

Achievable Diversity Orders of Decode-and-Forward Cooperative Protocols over Gamma-Gamma Fading FSO Links

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Abstract—In this paper, we consider the problem of relay-assisted free-space optical (FSO) transmissions with any number of relays. At a first time, we consider the quantum limited scenario and analyze the simple Decode-and-Forward (SDF) protocol where all symbols received at a certain relay are retransmitted to the destination. We also propose a novel Selective-DF (SelDF) protocol that can be implemented without requiring any kind of channel state information (CSI) neither at the transmitter nor at the receiver sides. We derive the diversity orders that can be achieved by SDF and SelDF over gamma-gamma fading channels and highlight the superiority of SelDF. We also prove that for some network configurations, SDF might achieve the same diversity order as SelDF. On the other hand, for most of the network configurations, the SDF protocol, that was studied extensively in the context of N_r -relays radio-frequency (RF) systems and one-relay FSO systems, is not suitable for FSO systems with more than one relay since it results in reduced diversity orders. At a second time, the above protocols are extended to FSO systems that are corrupted by background radiation. For SelDF, we propose a novel metric that captures the fidelity with which a certain message is detected at the relay thus determining whether this relay will participate in the cooperation effort or not.

Index Terms—Free-space optics, FSO, cooperation, relay, cooperative diversity, relay-assisted, fading channels.

I. INTRODUCTION

Recently, Free-Space Optical (FSO) communications attracted significant attention as a promising solution for the “last mile” problem [1]. In the same way as radio-frequency (RF) wireless communications, FSO links also suffer from fading. While fading in RF systems results from multi-path propagation, fading (or scintillation) in directive FSO links results from the variations of the index of refraction due to inhomogeneities in temperature and pressure changes [2]. Consequently, several fading-mitigation techniques that were initially proposed for RF communications were extended and tailored to FSO systems.

In the context of FSO communications, localized diversity techniques were studied in [3]–[10]. These techniques are based on deploying multiple apertures at the transmitter and/or receiver sides and they are analogous to the famous Multiple-Input-Multiple-Output (MIMO) RF systems. FSO localized diversity techniques include aperture-averaging receiver diversity [3], spatial repetition codes [4], [5], unipolar versions of the space-time codes [6]–[8] and transmit laser selection

[9], [10]. While MIMO-RF systems take advantage of the independence between the constituent channels to achieve high diversity and multiplexing gains, multiple-aperture FSO systems suffer from significant levels of channel correlation that follows from the high directivity of the FSO links. For example, the presence of a small cloud might induce large fades on all source-detector sub-channels simultaneously [4], thus rendering the high performance gains promised by MIMO-FSO systems difficult to achieve in practice. The above limitation motivated inspecting distributed diversity techniques, based on user cooperation, for FSO systems.

While the literature on cooperation in RF networks is huge and dates back to about a decade [11]–[13], it was only recently that cooperation started attracting a significant attention in the context of FSO systems [14]–[20]. The major difference between cooperative-RF and cooperative-FSO systems resides in the non-broadcast nature of FSO transmissions. While in RF networks a message sent from a source to a destination can be spontaneously overheard by neighboring nodes or relays, in FSO systems separate transceivers and a fraction of the available power must be entirely dedicated for delivering the information message to the relays. However, despite this power penalty, the various contributions in [14]–[20], highlighted the potential of cooperation as a powerful and practical tool for combatting fading in FSO systems.

The existing relay-assisted FSO systems can be classified into three categories. (i): Amplify-and-Forward (AF) relaying considered in [14] and [15] where in [14] N_r relays were deployed with no direct link between the source and destination while in [15] one relay was deployed assuming that a direct link is available between the source and destination. (ii): Decode-and-Forward (DF) relaying considered in [14], [16]–[19]. In [16], a cooperation strategy based on the implementation of convolutional codes was proposed and analyzed while in [17] a simple DF (SDF) strategy that can be implemented independently from the structure of the channel code was considered. Both strategies were considered in the context of 3-way cooperative FSO systems with a single relay and a direct link between the source and the destination. Selective DF (SelDF) protocols were proposed in [14] and [16] where a certain relay forwards the decoded message only if the received signal-to-noise ratio (SNR) or, in an equivalent manner, the path gain at this relay exceeds a certain threshold level. Under the quantum-limit scenario, it was proven in [19] that SDF and SelDF result in exactly the same performance with one relay. (iii): Relay-selection protocols where, based

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on the channel state, only a subset of the available relays participate in the cooperation effort. In this context, it was proven in [20] that the optimal power allocation strategy in the absence of background radiation consists of transmitting all data either along the direct link or along one of the indirect links via one relay that is appropriately chosen among all available relays. A similar relay-selection protocol that activates a single relay at a time was considered in [18] under the scenario of additive Gaussian noise.

In this paper, we analyze the SDF strategy with any number of relays (denoted by N_r) with intensity-modulation and direct-detection (IM/DD). We also present an alternative to the SelDF strategy considered in [14], [16]. In fact, the strategy in [14] and [16] is based on the SNR (or path gain) which is an average measure of the strength of the link between the source and a certain relay. In other words, for the same SNR, some symbols might be more efficiently reconstructed than others and the strategy in [14], [16] fails in distinguishing these symbols. Instead of being based on an average metric, the SelDF scheme that we propose is based on an alternative symbol-specific metric that measures the fidelity with which each particular symbol is recovered at the relay. This implies that even for the same channel realization, some symbols will be forwarded to the destination while others not (unlike [14], [16]). In this context, we propose two novel rules based on which the relay forwards the corresponding symbol or not. The first rule is suitable for the no-background radiation case and it does not require any kind of channel state information (CSI) while the second rule is applied in the presence of background radiation and it requires the estimation of the SNR at each relay.

In this work, we adopt the gamma-gamma fading channel model where the diversity order over the individual source-destination, source-relay and relay-destination FSO links depends on the distance of the link [21]. Based on an asymptotic analysis of the average error probability, we evaluate the diversity order that can be achieved by N_r -relay cooperative FSO networks implementing the SDF and SelDF strategies under the quantum limited scenario. In coherence with [21], the achievable diversity orders are highly dependent on the geometry of the network. For one-relay systems, our analysis shows that SDF and SelDF achieve the same diversity order. In this context, the systems considered in [19] (with $N_r = 1$ relay) over Rayleigh channels correspond only to a special case where the diversity orders over the intermediate links are the same (and equal to 1) and where the SDF and SelDF strategies achieve the full diversity order of two. With more than one relay, we prove the superiority of SelDF that is capable of achieving higher diversity orders compared to SDF. Based on the network configuration, the performance of SDF ranges from the best-case scenario, where SDF achieves exactly the same diversity order as SelDF, to the worst-case scenario where increasing the number of relays might not enhance the diversity order. While the diversity order of SelDF increases linearly with N_r for all network configurations, SDF manifests the same linear dependence under the above best-case scenario while the diversity order of SDF might sub-optimally scale as $\lceil \frac{N_r}{2} \rceil$ under the worst-case scenario.

Finally, since the proposed cooperation schemes are symbol-specific, an outage analysis is not appropriate for the problem under consideration. In fact, the outage probability is evaluated based on the assumption that a relay in outage will stop its transmission over the entire channel coherence time that extends over several thousands of symbol durations in the context of FSO communications. Consequently, the average error probability emerges as a more appropriate performance measure.

II. SYSTEM MODEL

We consider a relay-assisted FSO communication system where N_r neighboring nodes or relays share their optical transceivers for assisting a source S in delivering its message to a destination D. The relays, source and destination correspond to buildings on which FSO wireless units are deployed for establishing optical links with neighboring buildings. Since FSO links are highly directive, it is assumed that each relay possesses at least two separate transceivers; one transceiver is dedicated for the communication with S while the other one is deployed for establishing a link with D. In the same way, N_r transceivers are deployed on S (and on D) in order to set up optical connections with the relays. This configuration implies that the different signals received at D do not interfere with each other rendering all forms of parallel multiple access techniques or joint encoding/decoding unnecessary for cooperative FSO networks. The relays will be denoted by R_1, \dots, R_{N_r} in what follows. Because of the presence of these relays, the optical signal transmitted from S reaches D via the $N_r + 1$ parallel paths S-D, S- R_1 -D, \dots , S- R_{N_r} -D.

We denote by $I_0, I_{s,1}, \dots, I_{s,N_r}$ and $I_{1,d}, \dots, I_{N_r,d}$ the equivalent fading gains (irradiance fluctuations) through the optical channels between S-D, S- $R_1, \dots, S-R_{N_r}$ and R_1 -D, \dots, R_{N_r} -D, respectively. In the same way, we denote by $P_0, P_1^{(n)}$ and $P_2^{(n)}$ the fractions of the total power dedicated to the links S-D, S- R_n and R_n -D, respectively. These parameters must satisfy the relation $P_0 + \sum_{n=1}^{N_r} [P_1^{(n)} + P_2^{(n)}] = 1$ so that the cooperative system transmits the same total energy as non-cooperative systems.

In this work, we adopt the gamma-gamma turbulence-induced fading channel model [21] where the probability density function (pdf) of the irradiance ($I > 0$) is given by:

$$f(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{(\alpha+\beta)/2-1} K_{\alpha-\beta} \left(2\sqrt{\alpha\beta I} \right) \quad (1)$$

where $\Gamma(\cdot)$ is the Gamma function and $K_c(\cdot)$ is the modified Bessel function of the second kind of order c . The parameters α and β are given by:

$$\alpha(d) = \left[\exp \left(0.49\sigma_R^2(d)/(1 + 1.11\sigma_R^{12/5}(d))^{7/6} \right) - 1 \right]^{-1} \quad (2)$$

$$\beta(d) = \left[\exp \left(0.51\sigma_R^2(d)/(1 + 0.69\sigma_R^{12/5}(d))^{7/6} \right) - 1 \right]^{-1} \quad (3)$$

where $\sigma_R^2(d)$ is the Rytov variance related to the link distance d by:

$$\sigma_R^2(d) = 1.23C_n^2 k^{7/6} d^{11/6} \quad (4)$$

where k is the wave number and C_n^2 denotes the refractive index structure parameter [21].

Consider Q -ary pulse position modulation (PPM) with IM/DD. Denote by λ_s and λ_N the average number of photoelectrons per slot resulting from the incident light signal and background radiation (and/or dark currents), respectively. These parameters are given by [4]:

$$\lambda_s = \eta \frac{P_r T_s / Q}{hf} = \eta \frac{E_s}{hf} ; \quad \lambda_N = \eta \frac{P_b T_s / Q}{hf} \quad (5)$$

where T_s is the symbol duration and η is the detector's quantum efficiency assumed to be equal to 1 in what follows. $h = 6.6 \times 10^{-34}$ is Planck's constant and f is the optical center frequency taken to be 1.94×10^{14} Hz (corresponding to a wavelength of 1550 nm). P_r (resp. P_b) stands for the optical signal (resp. noise) power that is incident on the receiver. Finally, $E_s = P_r T_s / Q$ corresponds to the received optical energy per symbol along the direct link S-D.

As a first step in the proposed cooperation strategies, a PPM symbol $s \in \{1, \dots, Q\}$ is transmitted along the links S-D, S-R₁, ..., S-R_{N_r}. Denote by $\mathbf{Y}^{(0)}$ the Q -dimensional decision vector corresponding to the photoelectron counts (in the Q slots) at D. In the same way, denote by $\mathbf{Y}^{(n)}$ the decision vector at R _{n} for $n = 1, \dots, N_r$. In other words, $\mathbf{Y}^{(n)}$ can be written as $\mathbf{Y}^{(n)} = [Y_1^{(n)}, \dots, Y_Q^{(n)}]$ where $Y_q^{(n)}$ corresponds to the number of photoelectrons detected in the q -th slot along the link S-D for $n = 0$ and along the link S-R _{n} for $n = 1, \dots, N_r$.

For $q \neq s$, no light signal is transmitted in the q -th slot and the only source of photoelectrons in this slot is background radiation. In this case, $Y_q^{(n)}$ can be modeled as a Poisson random variable (r.v.) with parameter [4]:

$$\mathbb{E}[Y_q^{(n)}] = \lambda_N ; \quad q \neq s ; \quad n = 0, \dots, N_r \quad (6)$$

where $\mathbb{E}[\cdot]$ stands for the averaging operator.

For $q = s$, $Y_q^{(n)} = Y_s^{(n)}$ can be modeled as a Poisson r.v. with parameter:

$$\mathbb{E}[Y_s^{(n)}] = \begin{cases} P_0 I_0 \lambda_s + \lambda_N, & n = 0; \\ G_1^{(n)} P_1^{(n)} I_{s,n} \lambda_s + \lambda_N, & n = 1, \dots, N_r. \end{cases} \quad (7)$$

where $G_1^{(n)}$ is a gain factor that follows from the fact that S might be closer to R _{n} than it is to D. Performing a typical link budget analysis [14] shows that:

$$G_1^{(n)} = \left(\frac{d_{SD}}{d_{SR_n}} \right)^2 e^{-\zeta(d_{SR_n} - d_{SD})} \quad (8)$$

where ζ is the attenuation coefficient while d_{SD} and d_{SR_n} stand for the distances from S to D and from S to R _{n} , respectively, for $n = 1, \dots, N_r$.

Based on the decision vector $\mathbf{Y}^{(n)}$ available at R _{n} , the maximum-likelihood (ML) detector corresponds to deciding in favor of the symbol $\hat{s}^{(n)}$ given by:

$$\hat{s}^{(n)} = \arg \max_{q=1 \dots Q} Y_q^{(n)} ; \quad n = 1, \dots, N_r \quad (9)$$

where any tie in the above operation is broken randomly.

Now, the symbol $\hat{s}^{(n)}$ (rather than the symbol s) is transmitted along the link R _{n} -D. In this case, the corresponding decision vector at D can be written as $\mathbf{Z}^{(n)} = [Z_1^{(n)}, \dots, Z_Q^{(n)}]$

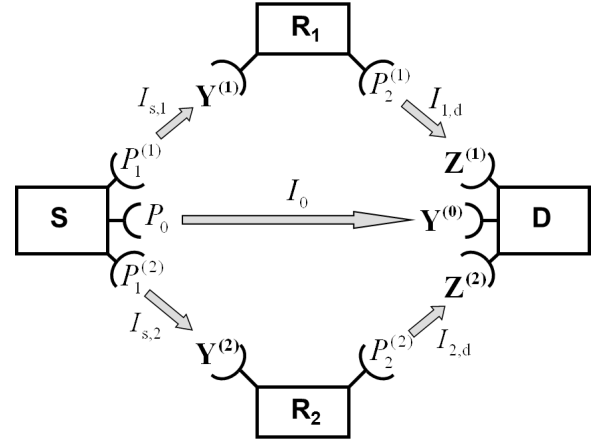


Fig. 1. Cooperation with two relays.

for $n = 1, \dots, N_r$ where $Z_q^{(n)}$ is a Poisson r.v. whose parameter is given by ($n = 1, \dots, N_r$):

$$\mathbb{E}[Z_q^{(n)}] = \begin{cases} G_2^{(n)} P_2^{(n)} I_{n,d} \lambda_s + \lambda_N, & q = \hat{s}^{(n)}; \\ \lambda_N, & q \neq \hat{s}^{(n)}. \end{cases} \quad (10)$$

where $G_2^{(n)} = \left(\frac{d_{SD}}{d_{R_n D}} \right)^2 e^{-\zeta(d_{R_n D} - d_{SD})}$ with $d_{R_n D}$ corresponding to the distance between R _{n} and D. The different parameters are depicted in Fig. 1 for $N_r = 2$ relays.

Finally, note that the synchronization problem, that arises since the relays are at different distances from D, falls beyond the scope of this paper and thus will not be considered. On the other hand, since the relays are stationary and the signals transmitted from these relays fall on separate transceivers at D, compensating for the relative delays between these signals seems to be less involved compared to cooperative RF systems.

III. COOPERATION IN THE ABSENCE OF BACKGROUND RADIATION

In this section, we consider the quantum-limit scenario where the impact of background radiation is removed by setting $\lambda_N = 0$. The quantum-limit scenario results in a simple analysis that offers clear and tractable insights on the fading mitigation capabilities of the cooperative system. This approach resembles the asymptotic analysis often adopted for the design of collocated and distributed RF diversity methods. Moreover, diversity methods result in the highest performance gains for high signal energies since they are designed to combat fading not noise. Under this scenario, the signal-dependent shot noise and fading become the main limiting factors on the performance since the power of background noise becomes very small compared to the power of the signal. In all circumstances, results obtained under the quantum-limit scenario can serve as lower bounds on the achievable performance levels in the presence of background radiation.

A. Cooperation strategies

We next propose two cooperation strategies that will be referred to as scheme 1 and scheme 2. For both schemes, a sequence of symbols is first transmitted simultaneously to

D and R_1, \dots, R_{N_r} . Given that $\lambda_N = 0$, therefore if symbol s was transmitted, then the decision variables $Y_q^{(n)}$ will be all equal to zero for $q \neq s$ and $n = 0, \dots, N_r$. Consequently, two scenarios are possible at the n -th relay R_n . (i): $Y_s^{(n)} > 0$; in this case, R_n decides in favor of $\hat{s}^{(n)} = s$ and the decision it makes is correct. In fact, in the absence of background radiation, the only source of this nonzero count in slot s is the presence of a light signal in this slot. (ii): $Y_s^{(n)} = 0$ implying that zero photoelectron counts are observed in all Q slots. In this case, the best that the relay R_n can do is to break the tie randomly and decide randomly in favor of one of the slots $\hat{s}^{(n)}$ resulting in a correct guess with probability $1/Q$.

For scheme 1, R_n forwards the decoded symbol $\hat{s}^{(n)}$ automatically to D independently from whether $Y_s^{(n)} > 0$ or $Y_s^{(n)} = 0$. This corresponds to the SDF protocol that was considered in [17] in the context of FSO transmissions with one relay. For scheme 2, R_n forwards the decoded symbol $\hat{s}^{(n)}$ only if $Y_s^{(n)} > 0$. In fact, if all counts are equal to zero ($Y_s^{(n)} = 0$), then most probably the relay R_n will make an erroneous decision (with probability $\frac{Q-1}{Q}$). In order to avoid confusing D by forwarding a wrong estimate of the symbol s , R_n backs off and stops its retransmission during the corresponding symbol duration. This scheme corresponds to a SelDF strategy where a relay participates in the cooperation effort only if a certain quality indicator is respected at this relay (in this case, the relay must observe one nonempty slot).

For both schemes, the decision at D will be based on the vectors $\mathbf{Y}^{(0)}$ and $\mathbf{Z}^{(1)}, \dots, \mathbf{Z}^{(N_r)}$. For scheme 1, the decoding strategy is as follows. If one slot of $\mathbf{Y}^{(0)}$ has a nonzero photoelectron count, then D decides in favor of this slot and its decision will be correct since symbol s is transmitted along the link S-D. On the other hand, if all components of $\mathbf{Y}^{(0)}$ are equal to zero, then D (i) inspects the vectors in $\{\mathbf{Z}^{(n)}\}_{n=1}^{N_r}$ having one nonzero component; (ii) determines the positions of these components and (iii) decides in favor of the position that is repeated the largest number of times. In a more formal way, denote by \mathcal{N} the set given by:

$$\mathcal{N} = \{n \mid \mathbf{Z}^{(n)} \neq \mathbf{0}_Q; n = 1, \dots, N_r\} \quad (11)$$

where $\mathbf{0}_Q$ corresponds to the Q -dimensional all-zero vector. Denote by $\tilde{s}^{(n)}$ the position of the nonzero component of $\mathbf{Z}^{(n)}$ for $n \in \mathcal{N}$:

$$\tilde{s}^{(n)} = \arg_{q=1, \dots, Q} [Z_q^{(n)} \neq 0] \quad ; \quad n \in \mathcal{N} \quad (12)$$

and construct the set \mathcal{S} as $\mathcal{S} = \{\tilde{s}^{(n)}; n \in \mathcal{N}\}$. In this case, D decides in favor of the element of \mathcal{S} that is repeated the most. In a more simplistic manner, when D receives no photoelectrons via the direct link S-D, it follows the decision made by the majority of the relays (if the number of photoelectrons received from these relays is nonzero). For example, if two (or more) relays are forwarding the same symbol, then most probably this symbol is the correct symbol since the probability of making an error at the relay is smaller than the probability of correct decision.

For scheme 2, the decoding rule is much simpler and D decides in favor of any non-empty slot of $\mathbf{Y}^{(0)}, \mathbf{Z}^{(1)}, \dots, \mathbf{Z}^{(N_r)}$.

In fact, the relays are all forwarding the correct symbol (otherwise they are backing off) and any photoelectron detected in any slot along the links S-D, S- R_1 -D, \dots , S- R_{N_r} -D will indicate correctly that the light signal was transmitted in this slot. Referring to the above formulation, $\tilde{s}^{(n)}$ will be equal to the transmitted symbol s for all values of n in \mathcal{N} . Note that for scheme 1, the case $\mathbf{Z}^{(n)} = \mathbf{0}_Q$ occurs only because of fading and shot noise along the link R_n -D while for scheme 2 this case might occur because R_n backed off as well.

To summarize, for both schemes, D decides in favor of the symbol \tilde{s} according to:

$$\tilde{s} = \begin{cases} \arg_{q=1, \dots, Q} [Y_q^{(0)} \neq 0], & \mathbf{Y}^{(0)} \neq \mathbf{0}_Q; \\ \arg \max_{q=1, \dots, Q} [\sum_{n \in \mathcal{N}} e_{\tilde{s}^{(n)}}], & \mathbf{Y}^{(0)} = \mathbf{0}_Q, \mathcal{N} \text{ nonempty}; \\ \text{rand}(1 \dots Q), & \mathbf{Y}^{(0)} = \mathbf{Z}^{(1)} = \dots = \mathbf{Z}^{(N_r)} = \mathbf{0}_Q. \end{cases} \quad (13)$$

where e_q stands for the q -th row of the $Q \times Q$ identity matrix and the function $\text{rand}(1, \dots, Q)$ corresponds to choosing randomly one integer in the set $\{1, \dots, Q\}$.

Note that a hard decision is made in (12) for the following reason. If the number of photoelectrons received via the link R_{n_1} -D is larger than that received via the link R_{n_2} -D (i.e. $Z_{\tilde{s}^{(n_1)}}^{(n_1)} > Z_{\tilde{s}^{(n_2)}}^{(n_2)}$), this does not mean that the decision vector $\mathbf{Z}^{(n_1)}$ is more reliable than the vector $\mathbf{Z}^{(n_2)}$. In fact, the value of the nonzero component of $\mathbf{Z}^{(n)}$ is related to the strength of the link R_n -D and is not in any manner related to the fidelity of the decision made at R_n . In other words, R_{n_1} might be forwarding a wrong symbol while R_{n_2} is forwarding the correct symbol. As a conclusion, if the hard decision in (12) was not performed, then a relay that is close to D and far from S will bias D to base its decision on the decision vector received from this relay thus degrading the performance of the system. As a conclusion, a simple equal gain combining (EGC) detection procedure is not appropriate for SDF since it does not take into consideration the fact that, unlike S, the relays might be forwarding wrong messages and since it does not perform hard decisions on the decision vectors received from the relays. Note that (13) is equivalent to EGC for SDF with $N_r = 1$ and for SelDF with any number of relays since in this case the relays (when active) are forwarding the correct symbol.

B. Performance Analysis

The channel state is defined by the vector $I \triangleq [I_0, I_{s,1}, \dots, I_{s,N_r}, I_{1,d}, \dots, I_{N_r,d}]$. For the sake of notational simplicity, we define the constants k_0 and $\{k_1^{(n)}, k_2^{(n)}\}_{n=1}^{N_r}$ as:

$$k_0 \triangleq P_0 I_0 \lambda_s; k_1^{(n)} \triangleq G_1^{(n)} P_1^{(n)} I_{s,n} \lambda_s; k_2^{(n)} \triangleq G_2^{(n)} P_2^{(n)} I_{n,d} \lambda_s \quad (14)$$

In what follows, $P_{e|I}^{(N_r)}$ stands for the conditional symbol-error probability (SEP) with N_r relays while $P_e^{(N_r)} \triangleq \int P_{e|I}^{(N_r)} p(I) dI$ stands for the SEP averaged over the gamma-gamma distributions of the components of I ($p(I)$ is the joint pdf of I).

In the absence of background radiation, the proposed cooperative schemes can be applied in the absence of CSI at the transmitter and receiver sides. In this case, no preference can be made among the $2N_r + 1$ links S-D, S- R_1 , \dots , S- R_{N_r} .

and R_1 -D, \dots , R_{N_r} -D and the best choice is to distribute the transmit power evenly among these links by setting:

$$P_0 = P_1^{(1)} = \dots = P_1^{(N_r)} = P_2^{(1)} = \dots = P_2^{(N_r)} = \frac{1}{2N_r + 1} \quad (15)$$

1) *Scheme 1*: We first analyze the performance of scheme 1. For $N_r = 1$ relay, the conditional SEP of scheme 1 was derived in [17] and it is given by:

$$P_{e|I}^{(1)} = \frac{Q-1}{Q} e^{-k_0} \left[e^{-k_1^{(1)}} + e^{-k_2^{(1)}} - e^{-k_1^{(1)}} e^{-k_2^{(1)}} \right] \quad (16)$$

We next determine the achievable diversity order based on an asymptotic analysis that holds for large values of λ_s . Asymptotically, the SEP is dominated by the behavior of the pdf near the origin where (1) can be approximated by [21]:

$$f(I) \approx a(d) I^{b(d)} \quad (17)$$

where $a(d)$ and $b(d)$ are related to the parameters in (2)-(3) by [21]:

$$a(d) = \frac{(\alpha(d)\beta(d))^{\beta(d)} \Gamma(\alpha(d) - \beta(d))}{\Gamma(\alpha(d)) \Gamma(\beta(d))} \quad (18)$$

$$b(d) = \beta(d) - 1 \quad (19)$$

In what follows, the following notations will be used: $a_0 \triangleq a(d_{SD})$, $a_1^{(n)} \triangleq a(d_{SR_n})$ and $a_2^{(n)} \triangleq a(d_{R_n D})$ in (18) and $\beta_0 \triangleq \beta(d_{SD})$, $\beta_1^{(n)} \triangleq \beta(d_{SR_n})$ and $\beta_2^{(n)} \triangleq \beta(d_{R_n D})$ in (3).

Averaging the conditional SEP in (16) over the approximate distributions (in (17)) of I_0 , $I_{s,1}$ and $I_{1,d}$ results in:

$$P_e^{(1)} = \frac{Q-1}{Q} \frac{a_0 \Gamma(\beta_0)}{(P_0 \lambda_s)^{\beta_0}} \left[\frac{a_1^{(1)} \Gamma(\beta_1^{(1)})}{(G_1^{(1)} P_1^{(1)} \lambda_s)^{\beta_1^{(1)}}} + \frac{a_2^{(1)} \Gamma(\beta_2^{(1)})}{(G_2^{(1)} P_2^{(1)} \lambda_s)^{\beta_2^{(1)}}} - \frac{a_1^{(1)} a_2^{(1)} \Gamma(\beta_1^{(1)}) \Gamma(\beta_2^{(1)})}{(G_1^{(1)} P_1^{(1)})^{\beta_1^{(1)}} (G_2^{(1)} P_2^{(1)})^{\beta_2^{(1)}} \lambda_s^{\beta_1^{(1)} + \beta_2^{(1)}}} \right] \quad (20)$$

Equation (20) scales asymptotically as $\lambda_s^{-(\beta_0 + \min\{\beta_1^{(1)}, \beta_2^{(1)}\})}$ implying that the diversity order that can be achieved by scheme 1 with one relay is $\beta_0 + \min\{\beta_1^{(1)}, \beta_2^{(1)}\}$ thus showing an enhancement with respect to non-cooperative systems where the achievable diversity order is β_0 . As indicated in [21], the diversity order depends on the positions of the relays since the parameters β_0 , $\beta_1^{(1)}$ and $\beta_2^{(1)}$ are distance-dependent.

In order to offer more insights on the properties of scheme 1, we first consider the special case of $N_r = 2$ before tackling the scenario with any number of relays. Assume that the symbol $s \in \{1, \dots, Q\}$ was transmitted. Denote by $p_e^{(n)}$ the conditional probability of error at relay R_n . An error occurs at this relay only when $Y_s^{(n)} = 0$; in this case, R_n makes a random decision among the Q slots resulting in:

$$p_e^{(n)} = \frac{Q-1}{Q} \Pr(Y_s^{(n)} = 0) = \frac{Q-1}{Q} e^{-k_1^{(n)}} \quad (21)$$

following from (7) and (14). On the other hand, a correct decision will be made at D when $Y_s^{(0)} > 0$. Consequently, the

conditional SEP can be written as:

$$P_{e|I}^{(2)} = \Pr(Y_s^{(0)} = 0) \left[\Pr(Z_{\hat{s}^{(1)}}^{(1)} = 0) \Pr(Z_{\hat{s}^{(2)}}^{(2)} = 0) p_{0,1}^{(2)} + \Pr(Z_{\hat{s}^{(1)}}^{(1)} > 0) \Pr(Z_{\hat{s}^{(2)}}^{(2)} = 0) p_{1,1}^{(2)} + \Pr(Z_{\hat{s}^{(1)}}^{(1)} = 0) \Pr(Z_{\hat{s}^{(2)}}^{(2)} > 0) p_{1,2}^{(2)} + \Pr(Z_{\hat{s}^{(1)}}^{(1)} > 0) \Pr(Z_{\hat{s}^{(2)}}^{(2)} > 0) p_{2,1}^{(2)} \right] \quad (22)$$

where $p_{i,j}^{(n)}$ is defined as the probability of error with n relays when nonzero photoelectron counts (at D) are observed from i relays. The integer j is introduced for indexing the possible choices of these i relays out of the n available relays ($j = 1, \dots, \binom{n}{i}$). In (22), $p_{0,1}^{(2)} = \frac{Q-1}{Q}$ since the case $Y_s^{(0)} = Z_{\hat{s}^{(1)}}^{(1)} = Z_{\hat{s}^{(2)}}^{(2)} = 0$ implies that $\mathbf{Y}^{(0)} = \mathbf{Z}^{(1)} = \mathbf{Z}^{(2)} = \mathbf{0}_Q$ resulting, from (13), in a random decision taken at D. On the other hand, $p_{1,1}^{(2)} = p_e^{(1)}$. In fact when $Y_s^{(0)} = 0$, $Z_{\hat{s}^{(2)}}^{(2)} = 0$ and $Z_{\hat{s}^{(1)}}^{(1)} > 0$, D will decide in favor of $\tilde{s} = \hat{s}^{(1)}$ since a nonzero photoelectron count was observed only in slot $\hat{s}^{(1)}$ (via the indirect link S- R_1 -D). This results in an erroneous decision ($\tilde{s} \neq s$) when an erroneous decision is made at R_1 ($\hat{s}^{(1)} \neq s$) with probability $p_e^{(1)}$. In the same way, $p_{1,2}^{(2)} = p_e^{(2)}$.

When $Y_s^{(0)} = 0$, $Z_{\hat{s}^{(1)}}^{(1)} > 0$ and $Z_{\hat{s}^{(2)}}^{(2)} > 0$, D will decide in favor of either $\tilde{s} = \hat{s}^{(1)}$ or $\tilde{s} = \hat{s}^{(2)}$. In fact, when $\hat{s}^{(1)} = \hat{s}^{(2)}$, the decision will be $\tilde{s} = \hat{s}^{(1)} = \hat{s}^{(2)}$. On the other hand, when $\hat{s}^{(1)} \neq \hat{s}^{(2)}$, D will decide randomly in favor of either $\tilde{s} = \hat{s}^{(1)}$ or $\tilde{s} = \hat{s}^{(2)}$. Consequently, $p_{2,1}^{(2)}$ can be written as:

$$p_{2,1}^{(2)} = (1 - p_e^{(1)})(1 - p_e^{(2)})q_0 + p_e^{(1)}p_e^{(2)}q_1 + (1 - p_e^{(1)})p_e^{(2)}q_2 + p_e^{(1)}(1 - p_e^{(2)})q_3 \quad (23)$$

where $q_0 = 0$ since for this event correct decisions are made at both relays resulting in $\hat{s}^{(1)} = s$ and $\hat{s}^{(2)} = s$ implying that D will decide correctly in favor of $\tilde{s} = s$. On the other hand, $q_1 = 1$ since for this event both relays made erroneous decisions. In this case, $\hat{s}^{(1)} \neq s$ and $\hat{s}^{(2)} \neq s$ and since D decides in favor of either $\tilde{s} = \hat{s}^{(1)}$ or $\tilde{s} = \hat{s}^{(2)}$, then in both cases $\tilde{s} \neq s$ and an error will be made at D. On the other hand, $q_2 = \frac{1}{2}$ since in this case $\hat{s}^{(1)} = s$ and $\hat{s}^{(2)} \neq s$ implying that D will make an erroneous decision with probability $\frac{1}{2}$. In the same way, $q_3 = \frac{1}{2}$. Now replacing $p_e^{(1)}$, $p_e^{(2)}$ and q_0, \dots, q_3 by their values in (23) and after some simplifications we obtain:

$$p_{2,1}^{(2)} = \frac{Q-1}{2Q} \left[e^{-k_1^{(1)}} + e^{-k_1^{(2)}} \right] \quad (24)$$

From (7) and (10): $\Pr(Y_s^{(0)} = 0) = e^{-k_0}$, $\Pr(Z_{\hat{s}^{(1)}}^{(1)} = 0) = 1 - \Pr(Z_{\hat{s}^{(1)}}^{(1)} > 0) = e^{-k_2^{(1)}}$ and $\Pr(Z_{\hat{s}^{(2)}}^{(2)} = 0) = 1 - \Pr(Z_{\hat{s}^{(2)}}^{(2)} > 0) = e^{-k_2^{(2)}}$. Replacing these probabilities as well as the values of $\{p_{0,1}^{(2)}, p_{1,1}^{(2)}, p_{1,2}^{(2)}, p_{2,1}^{(2)}\}$ in (22) results in:

$$P_{e|I}^{(2)} = e^{-k_0} \left[e^{-k_2^{(1)}} e^{-k_2^{(2)}} \frac{Q-1}{Q} + (1 - e^{-k_2^{(1)}}) e^{-k_2^{(2)}} \frac{Q-1}{Q} e^{-k_1^{(1)}} + e^{-k_2^{(1)}} (1 - e^{-k_2^{(2)}}) \frac{Q-1}{Q} e^{-k_1^{(2)}} + (1 - e^{-k_2^{(1)}})(1 - e^{-k_2^{(2)}}) \frac{Q-1}{2Q} \left(e^{-k_1^{(1)}} + e^{-k_1^{(2)}} \right) \right] \quad (25)$$

$$d_1 = \beta_0 + \min_{\substack{i \in \{1, \dots, N_r\} \\ i = N_r \text{ or } i \text{ even}}} \left\{ \min_{j=1, \dots, \binom{N_r}{i}} \left\{ \min_{n_1, \dots, n_{\lceil i/2 \rceil} \in \mathcal{A}_{i,j}} \left\{ \beta_1^{(n_1)} + \dots + \beta_1^{(n_{\lceil i/2 \rceil})} \right\} + \sum_{n \in \bar{\mathcal{A}}_{i,j}} \beta_2^{(n)} \right\} \right\} \triangleq \beta_0 + \min_{\substack{i \in \{1, \dots, N_r\} \\ i = N_r \text{ or } i \text{ even}}} \{d(i)\} \quad (28)$$

TABLE I
THE VALUES OF $d(i)$ FOR DIFFERENT NUMBER OF RELAYS

$N_r = 1$	$d(0)$	$\beta_2^{(1)}$	
	$d(1)$	$\beta_1^{(1)}$	
$N_r = 2$	$d(0)$	$\beta_2^{(1)} + \beta_2^{(2)}$	
	$d(2)$	$\min\{\beta_1^{(1)}, \beta_1^{(2)}\}$	
$N_r = 3$	$d(0)$	$\beta_2^{(1)} + \beta_2^{(2)} + \beta_2^{(3)}$	
	$d(2)$	$\min\{\min\{\beta_1^{(1)}, \beta_1^{(2)}\} + \beta_2^{(3)}, \min\{\beta_1^{(1)}, \beta_1^{(3)}\} + \beta_2^{(2)}, \min\{\beta_1^{(2)}, \beta_1^{(3)}\} + \beta_2^{(1)}\}$	
	$d(3)$	$\min\{\beta_1^{(1)} + \beta_1^{(2)}, \beta_1^{(1)} + \beta_1^{(3)}, \beta_1^{(2)} + \beta_1^{(3)}\}$	
$N_r = 4$	$d(0)$	$\beta_2^{(1)} + \beta_2^{(2)} + \beta_2^{(3)} + \beta_2^{(4)}$	
	$d(2)$	$\min\{\min\{\beta_1^{(1)}, \beta_1^{(2)}\} + \beta_2^{(3)} + \beta_2^{(4)}, \min\{\beta_1^{(1)}, \beta_1^{(3)}\} + \beta_2^{(2)} + \beta_2^{(4)}, \min\{\beta_1^{(1)}, \beta_1^{(4)}\} + \beta_2^{(2)} + \beta_2^{(3)}, \min\{\beta_1^{(2)}, \beta_1^{(3)}\} + \beta_2^{(1)} + \beta_2^{(4)}, \min\{\beta_1^{(2)}, \beta_1^{(4)}\} + \beta_2^{(1)} + \beta_2^{(3)}, \min\{\beta_1^{(3)}, \beta_1^{(4)}\} + \beta_2^{(1)} + \beta_2^{(2)}\}$	
	$d(4)$	$\min\{\min\{\beta_1^{(1)} + \beta_1^{(2)}\}, \min\{\beta_1^{(1)} + \beta_1^{(3)}\}, \min\{\beta_1^{(1)} + \beta_1^{(4)}\}, \min\{\beta_1^{(2)} + \beta_1^{(3)}\}, \min\{\beta_1^{(2)} + \beta_1^{(4)}\}, \min\{\beta_1^{(3)} + \beta_1^{(4)}\}\}$	

Averaging (25) over the channel vector I , results in:

$$P_e^{(2)} = \frac{Q-1}{Q} \frac{c_0}{\lambda_s^{\beta_0}} \left[\frac{c_2^{(1)} c_2^{(2)}}{\lambda_s^{\beta_2^{(1)} + \beta_2^{(2)}}} + \left(1 - \frac{c_2^{(1)}}{\lambda_s^{\beta_2^{(1)}}}\right) \frac{c_2^{(2)} c_1^{(1)}}{\lambda_s^{\beta_2^{(2)} + \beta_1^{(1)}}} + \left(1 - \frac{c_2^{(2)}}{\lambda_s^{\beta_2^{(2)}}}\right) \frac{c_2^{(1)} c_1^{(2)}}{\lambda_s^{\beta_2^{(1)} + \beta_1^{(2)}}} + \frac{1}{2} \left(1 - \frac{c_2^{(1)}}{\lambda_s^{\beta_2^{(1)}}}\right) \left(1 - \frac{c_2^{(2)}}{\lambda_s^{\beta_2^{(2)}}}\right) \left(\frac{c_1^{(1)}}{\lambda_s^{\beta_1^{(1)}}} + \frac{c_1^{(2)}}{\lambda_s^{\beta_1^{(2)}}}\right) \right] \quad (26)$$

where (for $m = 1, 2$ and $n = 1, \dots, N_r$):

$$c_0 \triangleq \frac{a_0 \Gamma(\beta_0)}{P_0^{\beta_0}} \quad ; \quad c_m^{(n)} \triangleq \frac{a_m^{(n)} \Gamma(\beta_m^{(n)})}{(G_m^{(n)} P_m^{(n)})^{\beta_m^{(n)}}} \quad (27)$$

Analyzing the asymptotic behavior of (26) shows that $P_e^{(2)} \rightarrow \lambda_s^{-d_1}$ for $\lambda_s \gg 1$ where the diversity gain of scheme 1 with two relays is: $d_1 = \beta_0 + \min\{\beta_1^{(1)}, \beta_1^{(2)}, \beta_2^{(1)} + \beta_2^{(2)}\}$.

For $N_r > 2$, the expressions of the conditional SEP become cumbersome. Moreover, since cooperation results in the highest performance gains for large values of λ_s , we further proceed with an asymptotic analysis that allows us to reach the following main result.

Proposition 1 [main result]: For cooperative FSO systems with N_r relays, scheme 1 (SDF relaying) achieves a diversity order of d_1 given in (28) at the top of the page. In (28), $\mathcal{A}_{i,1}, \dots, \mathcal{A}_{i,\binom{N_r}{i}}$ are all possible subsets of $\{1, \dots, N_r\}$ having i elements each while $\bar{\mathcal{A}}_{i,j} = \{1, \dots, N_r\} \setminus \mathcal{A}_{i,j}$.

Proof: The proof is provided in the appendix.

The values of $d(i)$ are summarized in Table-I for $N_r = 1, \dots, 4$. Note that since the parameters $\{\beta_1^{(n)}, \beta_2^{(n)}\}_{n=1}^{N_r}$ can take arbitrary values depending on the relay positions, further simplifications of the expression in (28) are not possible for the general case. Assume that all relays are at the same distance

from the source and from the destination resulting in $\beta_1^{(1)} = \dots = \beta_1^{(N_r)} \triangleq \beta_1$ and $\beta_2^{(1)} = \dots = \beta_2^{(N_r)} \triangleq \beta_2$. In this case, (28) simplifies to:

$$d_1 = \beta_0 + \min_{\substack{i \in \{1, \dots, N_r\} \\ i = N_r \text{ or } i \text{ even}}} \left\{ \left\lceil \frac{i}{2} \right\rceil \beta_1 + (N_r - i) \beta_2 \right\} \quad (29)$$

If the relays are much closer to S than they are to D, $\beta_1 \gg \beta_2$ resulting in $d_1 = \beta_0 + N_r \beta_2$ implying that, in this case, increasing the number of relays will always increase the diversity order. On the other hand, if the relays are much closer to D than they are to S, $\beta_2 \gg \beta_1$ resulting in $d_1 = \beta_0 + \lceil \frac{N_r}{2} \rceil \beta_1$ implying that increasing the number of relays, in this case, might not increase the achievable diversity order. For example, increasing the number of relays from 1 to 2 does not result in any enhancement in the diversity order. Finally, for $\beta_1 \approx \beta_2 \triangleq \beta$, $d_1 = \beta_0 + \lceil \frac{N_r}{2} \rceil \beta$. In the above second and third scenarios, it is always more advantageous to associate scheme 1 with an odd number of relays since the same diversity order can be achieved with a reduced system complexity.

2) *Scheme 2:* For scheme 2, an error occurs with probability $\frac{Q-1}{Q}$ (tie breaking) only when $\mathbf{Y}^{(0)} = \mathbf{Z}^{(1)} = \dots = \mathbf{Z}^{(N_r)} = \mathbf{0}_Q$. Consequently, the conditional SEP can be written as:

$$P_{e|I}^{(N_r)} = \frac{Q-1}{Q} \Pr(\mathbf{Y}^{(0)} = \mathbf{0}_Q) \prod_{n=1}^{N_r} \Pr(\mathbf{Z}^{(n)} = \mathbf{0}_Q) = \frac{Q-1}{Q} \Pr(\mathbf{Y}^{(0)} = \mathbf{0}_Q) \prod_{n=1}^{N_r} \left(1 - \Pr(\mathbf{Z}^{(n)} \neq \mathbf{0}_Q)\right) \quad (30)$$

Now $\mathbf{Z}^{(n)} \neq \mathbf{0}_Q$ if and only if R_n is not backing off (i.e. a nonzero photoelectron count was observed along the link S- R_n) and a nonzero photoelectron count was observed along the

link R_n -D. In other words, $\mathbf{Z}^{(n)} \neq \mathbf{0}_Q$ if and only if $Y_s^{(n)} > 0$ and $Z_{\hat{s}^{(n)}}^{(n)} > 0$. From (7) and (10), $\Pr(Y_s^{(n)} > 0) = 1 - e^{-k_1^{(n)}}$ and $\Pr(Z_{\hat{s}^{(n)}}^{(n)} > 0) = 1 - e^{-k_2^{(n)}}$ resulting in:

$$P_{e|I}^{(N_r)} = \frac{Q-1}{Q} e^{-k_0} \prod_{n=1}^{N_r} \left(e^{-k_1^{(n)}} + e^{-k_2^{(n)}} - e^{-k_1^{(n)}} e^{-k_2^{(n)}} \right) \quad (31)$$

Averaging the above probability results in:

$$P_e^{(N_r)} = \frac{Q-1}{Q} \frac{c_0}{\lambda_s^{\beta_0}} \prod_{n=1}^{N_r} \left[\frac{c_1^{(n)}}{\lambda_s^{\beta_1^{(n)}}} + \frac{c_2^{(n)}}{\lambda_s^{\beta_2^{(n)}}} - \frac{c_1^{(n)} c_2^{(n)}}{\lambda_s^{\beta_1^{(n)} + \beta_2^{(n)}}} \right] \quad (32)$$

which scales asymptotically as $\lambda_s^{-d_2}$ where the diversity order of scheme 2 is:

$$d_2 = \beta_0 + \sum_{n=1}^{N_r} \min\{\beta_1^{(n)}, \beta_2^{(n)}\} \quad (33)$$

Comparing the diversity orders achieved by scheme 1 and scheme 2 in (28) and (33), respectively, shows that increasing the number of relays always increases the value of d_2 while this is not the case for d_1 .

IV. COOPERATION IN THE PRESENCE OF BACKGROUND RADIATION

In this section, we consider the case where $\lambda_N \neq 0$. In this case, the background radiation results in nonzero photoelectron counts even in empty slots. As in the case of no background radiation, we consider two cooperative schemes: scheme 1 (SDF) where the relay decodes and forwards all symbols it receives without applying any selection process and scheme 2 (SelDF) where the relay backs off over some symbol durations in order not to confuse the destination with noisy replicas of the erroneous symbols it detected. Unlike the case of no background radiation where the relay R_n can be 100% sure that the symbol it detected is correct (when there is one nonempty slot in $\mathbf{Y}^{(n)}$), the presence of photoelectrons in empty slots due to background radiation imposes a certain level of uncertainty on the decision made at the relay.

For both schemes, the decision at D will be based on the vector $\mathbf{Z} = \mathbf{Y}^{(0)} + \sum_{n=1}^{N_r} \mathbf{Z}^{(n)}$ according to the rule $\tilde{s} = \arg \max_{q=1, \dots, Q} Z_q$ where Z_q stands for the q -th component of vector \mathbf{Z} . Because of the symmetry of the PPM constellation, we will evaluate the error performance assuming that the symbol $s = 1$ was transmitted. The parameters of the components of vector \mathbf{Z} that follow the Poisson distribution can be written as:

$$\Lambda_q \triangleq \mathbb{E}[Z_q] = \left[P_0 I_0 \delta_{q,1} + \sum_{n=1}^{N_r} \delta_{q, \hat{s}^{(n)}} G_2^{(n)} P_2^{(n)} I_{n,d} \right] \lambda_s + (N_r + 1) \lambda_N \quad ; \quad q = 1, \dots, Q \quad (34)$$

where $\delta_{i,j} = 1$ for $i = j$ and $\delta_{i,j} = 0$ for $i \neq j$. For scheme 1, the symbol $\hat{s}^{(n)}$ that is retransmitted from R_n is given in (9). For scheme 2, R_n stops its retransmission unless the probability of correct decision at R_n (denoted by $p_c^{(n)}$)

exceeds a certain threshold probability denoted by p_{th} . In this case, $\hat{s}^{(n)}$ can be written as:

$$\hat{s}^{(n)} = \begin{cases} \arg \max_{q=1, \dots, Q} Y_q^{(n)}, & p_c^{(n)} \geq p_{th}; \\ 0, & p_c^{(n)} < p_{th}. \end{cases} \quad (35)$$

where in the second case R_n is backing off and $\delta_{q, \hat{s}^{(n)}}$ will be equal to zero for $q \in \{1, \dots, Q\}$. Later in this section, we will show how to evaluate $p_c^{(n)}$. In what follows, scheme 1 will be treated as a special case of scheme 2 by setting $p_{th} = 0$.

Conditioned on the channel state vector I and on the decisions made at the relays $\hat{\mathbf{s}} \triangleq (\hat{s}^{(1)}, \dots, \hat{s}^{(N_r)})$, the conditional SEP can be written as:

$$P_{e|I, \hat{\mathbf{s}}}^{(N_r)} = 1 - P_{c|I, \hat{\mathbf{s}}}^{(N_r)} \leq 1 - \prod_{q=2}^Q \Pr(Z_q < Z_1) \quad (36)$$

$$= 1 - \prod_{q=2}^Q \left[\sum_{k=0}^{+\infty} \Pr(Z_1 = k) \Pr(Z_q < k) \right] \quad (37)$$

$$= 1 - \prod_{q=2}^Q \left[\sum_{k=0}^{+\infty} \frac{e^{-\Lambda_1} \Lambda_1^k}{k!} \sum_{j=0}^{k-1} \frac{e^{-\Lambda_q} \Lambda_q^j}{j!} \right] \quad (38)$$

where the inequality in (36) follows from ignoring the probability of correct decision when ties occur while (38) follows from (34). Averaging over $\hat{\mathbf{s}} \in \{0, 1, \dots, Q\}^{N_r}$ results in:

$$P_{e|I}^{(N_r)} \leq 1 - \sum_{(c_1, \dots, c_{N_r}) \in \{0, 1, \dots, Q\}^{N_r}} \prod_{n=1}^{N_r} \Pr(\hat{s}^{(n)} = c_n) \prod_{q=2}^Q \left[\sum_{k=0}^{+\infty} \frac{e^{-\Lambda_1} \Lambda_1^k}{k!} \sum_{j=0}^{k-1} \frac{e^{-\Lambda_q} \Lambda_q^j}{j!} \right] \quad (39)$$

where:

$$\Pr(\hat{s}^{(n)} = c_n) = \Pr(p_c^{(n)} < p_{th}) \quad ; \quad c_n = 0 \quad (40)$$

$$\Pr(\hat{s}^{(n)} = c_n) \leq \Pr(p_c^{(n)} \geq p_{th}) \prod_{q=1; q \neq c_n}^Q \Pr(Y_q^{(n)} < Y_{c_n}^{(n)}) \quad ; \quad c_n \neq 0 \quad (41)$$

and the inequality in (41) follows from assuming that R_n always decides in favor of $\hat{s}^{(n)} \neq c_n$ when ties occur. For scheme 1, $\Pr(p_c^{(n)} \geq p_{th}) = \Pr(p_c^{(n)} \geq 0) = 1$ and $\Pr(p_c^{(n)} < p_{th}) = 0$.

On the other hand, under the assumption that the symbol $s = 1$ was transmitted, the parameters of the components of $\mathbf{Y}^{(n)}$ that follow the Poisson distribution are given by:

$$\Lambda_q^{(n)} \triangleq \mathbb{E}[Y_q^{(n)}] = G_1^{(n)} P_1^{(n)} I_{s,n} \delta_{q,1} \lambda_s + \lambda_N \quad ; \quad q = 1, \dots, Q \quad (42)$$

Finally, combining the results obtained in (39)-(42) results in (43) shown at the top of the next page.

The above expression does not lend itself to a simple mathematical analysis and it does not result in a closed-form solution for $P_e^{(N_r)}$. Consequently, unlike the case $\lambda_N = 0$, an expression for the diversity order can not be reached in the case $\lambda_N \neq 0$. In this case, the diversity orders obtained in section III in the absence of background radiation constitute

$$P_{e|I}^{(N_r)} \leq 1 - \sum_{(c_1, \dots, c_{N_r}) \in \{0,1, \dots, Q\}^{N_r}} \prod_{n=1}^{N_r} \left[\delta_{c_n,0} \Pr(p_c^{(n)} < p_{th}) + (1 - \delta_{c_n,0}) \Pr(p_c^{(n)} \geq p_{th}) \right] \prod_{q'=1}^Q \left[\sum_{k=0}^{+\infty} \frac{e^{-\Lambda_{c_n}^{(n)}} (\Lambda_{c_n}^{(n)})^k}{k!} \sum_{j=0}^{k-1} \frac{e^{-\Lambda_{q'}^{(n)}} (\Lambda_{q'}^{(n)})^j}{j!} \right] \prod_{q=2}^Q \left[\sum_{k=0}^{+\infty} \frac{e^{-\Lambda_1} \Lambda_1^k}{k!} \sum_{j=0}^{k-1} \frac{e^{-\Lambda_q} \Lambda_q^j}{j!} \right] \quad (43)$$

lower bounds on the diversity orders that can be achieved by SDF and SelDF in the presence of background radiation.

We next evaluate $p_c^{(n)}$ that constitutes a metric based on which R_n will forward the message or not. Assume that R_n decided in favor of $\hat{s}^{(n)}$ according to (9). This decision is correct (i.e. $\hat{s}^{(n)}$ is indeed the symbol that was transmitted) with probability $p_c^{(n)}$ that can be written as:

$$\begin{aligned} p_c^{(n)} &= \Pr(\hat{s}^{(n)} \text{ transmitted} | \mathbf{Y}^{(n)}) \\ &= \frac{\Pr(\mathbf{Y}^{(n)} | \hat{s}^{(n)} \text{ transmitted}) \Pr(\hat{s}^{(n)} \text{ transmitted})}{\Pr(\mathbf{Y}^{(n)})} \\ &= \frac{\Pr(\mathbf{Y}^{(n)} | \hat{s}^{(n)} \text{ transmitted}) \Pr(\hat{s}^{(n)} \text{ transmitted})}{\sum_{q=1}^Q \Pr(q \text{ transmitted}) \Pr(\mathbf{Y}^{(n)} | q \text{ transmitted})} \quad (44) \end{aligned}$$

where $\Pr(\hat{s}^{(n)} \text{ transmitted}) = \Pr(q \text{ transmitted}) = \frac{1}{Q}$ since all elements of $\{1, \dots, Q\}$ are transmitted with the same probability. On the other hand, when symbol q is transmitted, then the parameters of the components of $\mathbf{Y}^{(n)}$ are given by (symbol q transmitted):

$$E[Y_{q'}^{(n)}] = \begin{cases} G_1^{(n)} P_1^{(n)} I_{s,n} \lambda_s + \lambda_N = k_1^{(n)} + \lambda_N, & q' = q; \\ \lambda_N, & q' \neq q. \end{cases} \quad (45)$$

where $k_1^{(n)}$ is defined in (14). Now, (44) can be written as:

$$p_c^{(n)} = \frac{\frac{e^{-(k_1^{(n)} + \lambda_N)} (k_1^{(n)} + \lambda_N)^{Y_{\hat{s}^{(n)}}^{(n)}}}{Y_{\hat{s}^{(n)}}^{(n)}!} \prod_{q \neq \hat{s}^{(n)}}^Q \frac{e^{-\lambda_N} \lambda_N^{Y_q^{(n)}}}{Y_q^{(n)}!}}{\sum_{q=1}^Q \frac{e^{-(k_1^{(n)} + \lambda_N)} (k_1^{(n)} + \lambda_N)^{Y_q^{(n)}}}{Y_q^{(n)}!} \prod_{q' \neq q}^Q \frac{e^{-\lambda_N} \lambda_N^{Y_{q'}^{(n)}}}{Y_{q'}^{(n)}!}} \quad (46)$$

After some manipulations, (46) simplifies to:

$$p_c^{(n)} = \frac{\left(1 + \frac{k_1^{(n)}}{\lambda_N}\right)^{Y_{\hat{s}^{(n)}}^{(n)}}}{\sum_{q=1}^Q \left(1 + \frac{k_1^{(n)}}{\lambda_N}\right)^{Y_q^{(n)}}} = \frac{\left(1 + \frac{k_1^{(n)}}{\lambda_N}\right)^{\max\{Y_1^{(n)}, \dots, Y_Q^{(n)}\}}}{\sum_{q=1}^Q \left(1 + \frac{k_1^{(n)}}{\lambda_N}\right)^{Y_q^{(n)}}} \quad (47)$$

Note that the relation $p_c^{(n)} \geq p_{th}$ does not guarantee that $\hat{s}^{(n)} = s$; however, it imposes some kind of selectivity on the symbols to be forwarded by R_n . Finally, note that the evaluation of (47) at a certain relay R_n requires estimating the SNR (given by $\frac{k_1^{(n)}}{\lambda_N}$) at this relay. Consequently, unlike the no-background radiation case where scheme 2 can be implemented in the absence of CSI, the relay R_n must estimate the parameters of the link S- R_n in the presence of background radiation. In this case, no CSI needs to be acquired by D. On the other hand, the relay-selection protocols (for example [20]) achieve higher performance levels at the expense of

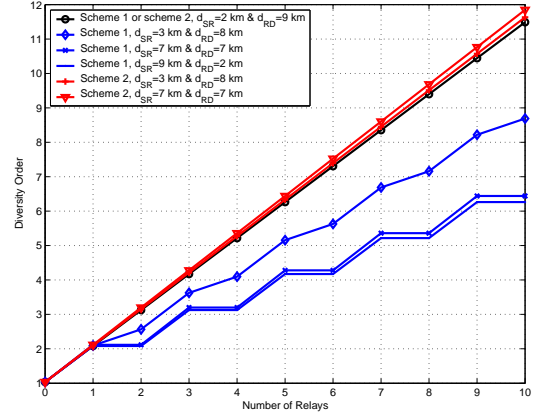


Fig. 2. The achievable diversity orders by scheme 1 and scheme 2 in the absence of background radiation.

an increased complexity. In fact, the implementation of these protocols requires estimating the parameters of the $2N_r + 1$ links S-D, S- $R_1, \dots, S-R_{N_r}$, R_1 -D, \dots , R_{N_r} -D. Moreover, unlike scheme 2 where each relay decides by itself whether it will cooperate or not, a feedback mechanism needs to be implemented in [20] in order to inform every relay whether it will be active or not since the relays can no longer make these decisions by themselves independently from the underlying state of the entire network.

V. NUMERICAL RESULTS

We next present some numerical results that support the theoretical claims made in the previous sections. For simulation purposes, we assume that all relays are at the same distance from the source and the destination resulting in $d_{SR_1} = \dots = d_{SR_{N_r}} \triangleq d_{SR}$ and $d_{R_1D} = \dots = d_{R_{N_r}D} \triangleq d_{RD}$. The distance between S and D is fixed to $d_{SD} = 10$ km and we set $C_n^2 = 1.7 \times 10^{-14} \text{ m}^{-2/3}$ and $\zeta = 0.43 \text{ dB/km}$ in (4) and (8), respectively.

Fig. 2 shows the diversity orders that can be achieved by scheme 1 and scheme 2 as given in (29) and (33), respectively. This figure shows that the achievable diversity orders are highly dependent on the relay positions. Moreover, while the diversity order of scheme 2 increases linearly with the number of relays, the diversity order of scheme 1 exhibits a linear increase for $(d_{SR}, d_{RD}) = (2, 9)$ km ($\beta_1 \gg \beta_2$) and a stair-like increase for $(d_{SR}, d_{RD}) = (9, 2)$ km ($\beta_1 \ll \beta_2$) and $(d_{SR}, d_{RD}) = (7, 7)$ km ($\beta_1 = \beta_2$). A mixture of these two extreme variations is observed for $(d_{SR}, d_{RD}) = (3, 8)$ km where none of the relations $\beta_1 \gg \beta_2$, $\beta_1 \ll \beta_2$ or $\beta_1 = \beta_2$ is satisfied. For scheme 2, the relay positions satisfying

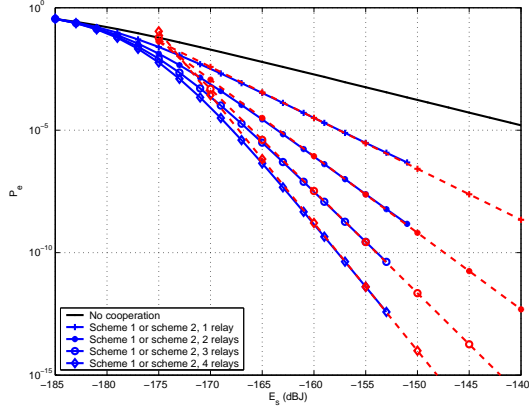


Fig. 3. Performance of 4-PPM in the absence of background radiation for $d_{SR} = 2$ km and $d_{RD} = 9$ km. Solid and dashed lines correspond to the exact error probabilities and bounds, respectively.

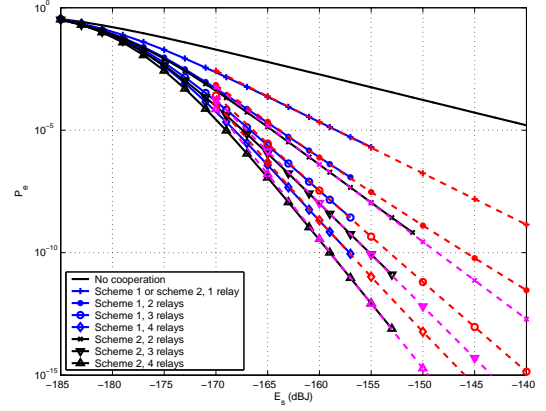


Fig. 5. Performance of 4-PPM in the absence of background radiation for $d_{SR} = 3$ km and $d_{RD} = 8$ km. Solid and dashed lines correspond to the exact error probabilities and bounds, respectively.

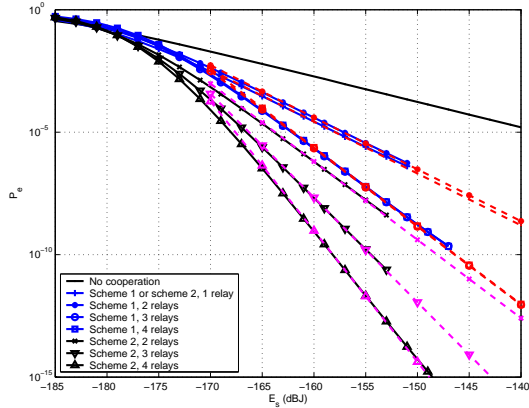


Fig. 4. Performance of 4-PPM in the absence of background radiation for $d_{SR} = d_{RD} = 7$ km. Solid and dashed lines correspond to the exact error probabilities and bounds, respectively.

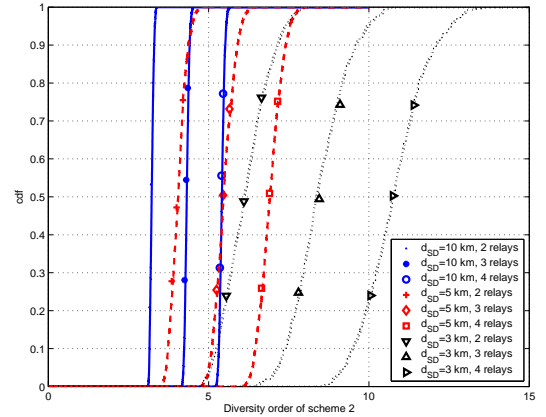


Fig. 6. cdf of d_2 for random relay positions.

$(d_{SR}, d_{RD}) = (d_1, d_2)$ and $(d_{SR}, d_{RD}) = (d_2, d_1)$ will result in the same diversity order where d_1 and d_2 are arbitrary distances. For scheme 1, the position $(d_{SR}, d_{RD}) = (d_1, d_2)$ where $d_1 < d_2$ results in a higher diversity order compared to the position $(d_{SR}, d_{RD}) = (d_2, d_1)$ as shown in Fig. 2 for the cases $(d_{SR}, d_{RD}) = (2, 9)$ km and $(d_{SR}, d_{RD}) = (9, 2)$ km. In other words, for scheme 1 it is always more advantageous to place the relays closer to S.

Figures 3, 4 and 5 compare the performance of scheme 1 and scheme 2 with 4-PPM in the absence of background radiation for $(d_{SR}, d_{RD}) = (2, 9)$ km, $(d_{SR}, d_{RD}) = (7, 7)$ km and $(d_{SR}, d_{RD}) = (3, 8)$ km, respectively. All results show the extremely close match (for large values of E_s) between the exact error probabilities and the upper-bounds obtained from approximating the gamma-gamma pdf by (17). More precisely, the exact error probabilities tend to the upper-bounds as E_s increases showing that these bounds can accurately predict the achievable diversity orders. In coherence with the results in Fig. 2, Fig. 3 shows that scheme 1 and scheme 2 achieve exactly the same performance level for $(d_{SR}, d_{RD}) = (2, 9)$ km. In this scenario, very high performance gains can be obtained. For example, while non-cooperative systems have

a diversity order of 1.035, deploying scheme 1 or scheme 2 with four relays increases the diversity order to 5.2 resulting in a huge performance gain of 29 dB at $P_e = 10^{-5}$. On the other hand, Fig. 4 shows the huge gap between the two considered schemes in the scenario where $(d_{SR}, d_{RD}) = (7, 7)$ km. In this case, for scheme 1, increasing the number of relays from 3 to 4 does not enhance the performance level while increasing the number of relays from 1 to 2 even deteriorates the performance by about 0.7 dB. In both cases, the increase in the number of relays does not affect the diversity order as predicted from Fig. 2. Finally, results in Fig. 5 show that the performance of scheme 1 improves with the number of relays without exceeding the performance levels achieved by scheme 2. Finally, results in figures 3, 4 and 5 show the importance of implementing a SelDF protocol at the relays.

While the above figures showed the performance for fixed relay positions with $d_{SD} = 10$ km, Fig. 6 shows the achievable diversity orders by scheme 2 for random relay positions with $d_{SD} = 3$ km, $d_{SD} = 5$ km and $d_{SD} = 10$ km. In this case, β_0 that quantifies the degree of fading along the direct link takes the values of 1.53, 1.17 and 1.03, respectively. Results show the cumulative distribution function (cdf) of d_2 in (33) for different number of relays where the positions of the relays are generated randomly inside the circle of diameter

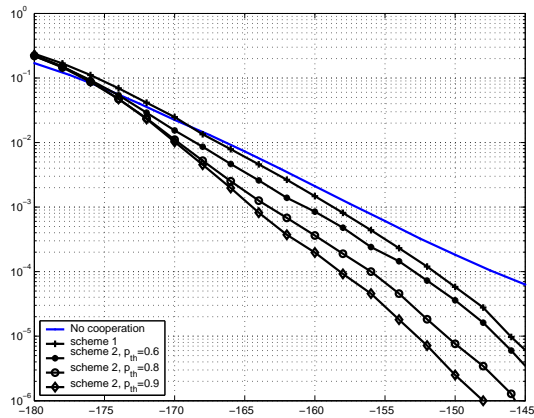


Fig. 7. Performance of 2-PPM in the presence of background radiation with $P_b T_s/Q = -185$ dBJ and $d_{SR} = d_{RD} = 7$ km. Scheme 1 and scheme 2 are deployed with one relay.

SD. Results show the critical effect of the link distance on the performance. For example, 3-km transmissions assisted by two relays outperform 5-km transmissions assisted by three relays.

Fig. 7 shows the performance of 2-PPM with $N_r = 1$ relay in the presence of background radiation where we fix $P_b T_s/Q = -185$ dBJ and $d_{SR} = d_{RD} = 7$ km. This figure compares the performance of non-cooperative systems with that of scheme 1 and scheme 2 for different values of the threshold probability p_{th} . This figure shows that scheme 2 outperforms scheme 1 for $p_{th} = 0.6, \dots, 0.9$ and that the performance gains increase with p_{th} . Moreover, for scheme 1, cooperation becomes useful for the values of E_s exceeding -169 dBJ while for scheme 2 this value is reduced to about -175 dBJ.

Fig. 8 shows the performance of scheme 2 with 2-PPM for various values of N_r . In this figure, we fix $P_b T_s/Q = -185$ dBJ, $p_{th} = 0.9$, $d_{SR} = 2$ km and $d_{RD} = 9$ km. For large values of E_s , results show the improvements in the performance levels and diversity orders as the number of relays increases. Huge performance gains exceeding 12 dB can be observed with 4 relays at a SEP of 10^{-3} . The numerical results show that the diversity orders are 2.025, 3.226 and 4.123 with 1, 2 and 3 relays, respectively. On the other hand, results in Fig. 2 show that the diversity orders, that are derived asymptotically in the absence of background radiation, are 2.116, 3.197 and 4.279, respectively. The above values are very close showing that the derived diversity orders are also useful for systems corrupted by background noise.

VI. CONCLUSION

We investigated SDF and SelDF as candidate solutions for relay-assisted FSO communication systems with any number of relays. The theoretical analysis and the numerical results showed that, in general, the SelDF protocol is capable of achieving higher diversity orders than the SDF protocol where the diversity order of SDF is at best equal to that of SelDF. In the absence of background radiation, the superiority of SelDF is not associated with any increase in the system complexity since both SDF and SelDF can be implemented in the absence of both CSI and feedback. In the presence of background

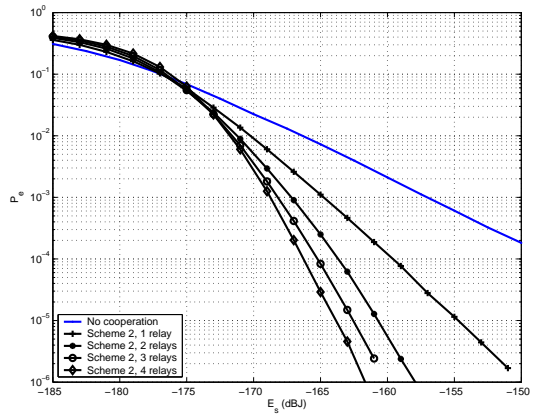


Fig. 8. Performance of scheme 2 with 2-PPM in the presence of background radiation with $p_{th} = 0.9$, $P_b T_s/Q = -185$ dBJ, $d_{SR} = 2$ km and $d_{RD} = 9$ km.

radiation, SelDF results in a more sophisticated relay structure since each relay now needs to estimate the SNR of the link it establishes with the source. In this scenario, the complexities of the source and the destination are the same with SDF and SelDF. As in the no-background radiation case, SelDF does not require any kind of feedback.

APPENDIX

The integral of a term having the form $e^{-k_m^{(n)}} = e^{-G_m^{(n)} P_m^{(n)} I \lambda_s}$ over the approximate gamma-gamma distribution in (17) of the irradiance I results in $\frac{a_m^{(n)} \Gamma(\beta_m^{(n)})}{(G_m^{(n)} P_m^{(n)} \lambda_s) \beta_m^{(n)}}$ which scales asymptotically as $\lambda_s^{-\beta_m^{(n)}}$ where $\beta_m^{(n)}$ is the parameter of the gamma-gamma distribution given in (3). Based on this observation, the following notation will be used in this appendix:

$$f(\lambda_s) \doteq d \quad \text{means} \quad \lim_{\lambda_s \rightarrow +\infty} \frac{\log \int f(\lambda_s) p(I) dI}{\log \lambda_s} = -d \quad (48)$$

For example, $p_{2,1}^{(2)} \doteq \min\{\beta_1^{(1)}, \beta_1^{(2)}\}$ in (24). Extending (22), the conditional SEP of scheme 1 with N_r relays can be written as:

$$P_{e|I}^{(N_r)} = e^{-k_0} \sum_{i=0}^{N_r} \sum_{j=1}^{N_r} P_{i,j}^{(N_r)} \Pi_1 \Pi_2 \quad (49)$$

$$\triangleq e^{-k_0} \sum_{i=0}^{N_r} \sum_{j=1}^{N_r} P_{i,j}^{(N_r)} \prod_{m_1 \in \mathcal{I}_{i,j}(\{1, \dots, N_r\})} (1 - e^{-k_2^{(m_1)}}) \prod_{m_2 \in \{1, \dots, N_r\} \setminus \mathcal{I}_{i,j}(\{1, \dots, N_r\})} e^{-k_2^{(m_2)}} \quad (50)$$

where the sets $\mathcal{I}_{i,1}(\mathcal{A}), \dots, \mathcal{I}_{i,|\mathcal{A}|}(\mathcal{A})$ are all possible subsets of the set \mathcal{A} having i elements each. For example, for $N_r = 3$, $\mathcal{I}_{2,1}(\{1, 2, 3\}) = \{1, 2\}$, $\mathcal{I}_{2,2}(\{1, 2, 3\}) = \{1, 3\}$ and $\mathcal{I}_{2,3}(\{1, 2, 3\}) = \{2, 3\}$. Evidently, $\mathcal{I}_{0,1}(\{1, \dots, N_r\})$ is empty and $\mathcal{I}_{N_r,1}(\{1, \dots, N_r\}) = \{1, \dots, N_r\}$.

In (50), the integer i stands for the number of relays from which D observes non-zero photoelectron counts while the integer j indexes the possible choices of these i relays out

of the N_r available relays. In this case, the error probability $P_{i,j}^{(N_r)}$ can be written as:

$$P_{i,j}^{(N_r)} = \sum_{i'=0}^i \sum_{j'=1}^{(i')} p_{i,j,i',j'}^{(N_r)} \Pi_1' \Pi_2' \quad (51)$$

$$\triangleq \sum_{i'=0}^i \sum_{j'=1}^{(i')} p_{i,j,i',j'}^{(N_r)} \prod_{n_1 \in \mathcal{I}_{i',j'}(\mathcal{I}_{i,j}(\{1, \dots, N_r\}))} p_e^{(n_1)} \prod_{n_2 \in \mathcal{I}_{i,j}(\{1, \dots, N_r\}) \setminus \mathcal{I}_{i',j'}(\mathcal{I}_{i,j}(\{1, \dots, N_r\}))} (1 - p_e^{(n_2)}) \quad (52)$$

where the integer i' stands for the number of relays that are making erroneous decisions while the integer j' indexes the possible choices of these i' relays out of the i relays that result in nonzero counts at D. In (52), $p_e^{(m)}$ is the error probability at the m -th relay given in (21). Based on (13), the majority choice will be made exclusively among the i relays that result in nonzero photoelectron counts at D while the remaining $N_r - i$ relays will be ignored in the decision process. In other words, D receives nothing from these $N_r - i$ relays and it proceeds very simply as if they do not exist.

In (52), the probability $p_{i,j,i',j'}^{(N_r)}$ stands for the error probability when D receives nonzero counts from i relays among which i' relays are forwarding erroneous messages. Consequently, $p_{i,j,i',j'}^{(N_r)}$ depends on the manner in which the majority among the i relays is selected. Therefore, $p_{i,j,i',j'}^{(N_r)}$ is a function of N_r , Q , i and i' and it does not depend on any fading gain among $I_0, I_{s,1}, \dots, I_{s,N_r}, I_{1,d}, \dots, I_{N_r,d}$. As a conclusion, $p_{i,j,i',j'}^{(N_r)} \doteq 0$. Evidently, $p_{i,j,i',j'}^{(N_r)}$ does not depend on j and j' that are used for enumerating all possible combinations. For example, let us evaluate $p_{3,j,2,j'}^{(3)}$ where 1 relay is forwarding a correct symbol in slot s while the remaining 2 relays are forwarding erroneous symbols in the two slots $s', s'' \in \{1, \dots, s-1, s+1, \dots, Q\}$. In this case, two scenarios are possible. (i): The two erroneous symbols happen to be the same (i.e. $s' = s''$ with probability $\frac{1}{Q-1}$). In this case, D will decide in favor of the majority $\bar{s} = s' = s''$ resulting in an error with probability 1. (ii): $s' \neq s''$ with probability $\frac{Q-2}{Q-1}$. In this case, D makes a random choice among the 3 different values s, s' and s'' resulting in an erroneous decision with probability $2/3$. Consequently, $p_{3,j,2,j'}^{(3)} = 1 \frac{1}{Q-1} + \frac{2}{3} \frac{Q-2}{Q-1} = \frac{2Q-1}{3(Q-1)}$. In general, a closed-form general expression of $p_{i,j,i',j'}^{(N_r)}$ can not be reached and the evaluation of this probability becomes tedious for large values of N_r . However, no matter what the value of this probability is, it has no impact on the diversity order of the system and hence the asymptotic analysis provided in this appendix is based on $p_{i,j,i',j'}^{(N_r)} \doteq 0$ independently from the specific value of this error probability.

Since in (52), i' relays are forwarding erroneous symbols while $i - i'$ relays are forwarding the correct symbol, then a correct decision can be guaranteed at D (that is making a majority decision) when $i' < i - i'$ or equivalently when $i' \leq \lfloor i/2 \rfloor$. In other words, $p_{i,j,i',j'}^{(N_r)} = 0$ for $i' \leq \lfloor i/2 \rfloor$ and

(52) can be written as:

$$P_{i,j}^{(N_r)} = \sum_{i'=\lfloor i/2 \rfloor}^i p_{i,i'}^{(N_r)} \sum_{j'=1}^{(i')} \prod_{n_1 \in \mathcal{I}_{i',j'}(\mathcal{I}_{i,j}(\{1, \dots, N_r\}))} p_e^{(n_1)} \prod_{n_2 \in \mathcal{I}_{i,j}(\{1, \dots, N_r\}) \setminus \mathcal{I}_{i',j'}(\mathcal{I}_{i,j}(\{1, \dots, N_r\}))} (1 - p_e^{(n_2)}) \quad (53)$$

where $p_{i,j,i',j'}^{(N_r)}$ was written as $p_{i,i'}^{(N_r)}$ since it does not depend on j and j' .

From (51) and (53), $\Pi_2' \doteq 0$ while $\Pi_1' \doteq \sum_{n_1 \in \mathcal{I}_{i',j'}(\mathcal{I}_{i,j}(\{1, \dots, N_r\}))} \beta_1^{(n_1)}$ where this summation contains i' summands. Since $i' \in \{\lfloor i/2 \rfloor, \dots, i\}$, then the diversity order of Π_1' is obtained when i' takes its minimum possible value of $\lfloor i/2 \rfloor$. In fact, for $i' > \lfloor i/2 \rfloor$ the last summation contains additional terms compared to the case $i' = \lfloor i/2 \rfloor$ and hence the corresponding power of λ_s^{-1} will increase implying that asymptotically the smaller power of λ_s^{-1} obtained for $i' = \lfloor i/2 \rfloor$ will dominate. As a conclusion, the summation over i' can be removed from (53) and i' can be replaced by $\lfloor i/2 \rfloor$. Consequently, the diversity order of (53) can be written as:

$$P_{i,j}^{(N_r)} \doteq \min_{j'=1, \dots, \binom{i}{\lfloor i/2 \rfloor}} \sum_{n_1 \in \mathcal{I}_{\lfloor i/2 \rfloor, j'}(\mathcal{I}_{i,j}(\{1, \dots, N_r\}))} \beta_1^{(n_1)} \quad (54)$$

since $p_{i,i'}^{(N_r)} \doteq 0$.

The summation in (54) contains $\lfloor i/2 \rfloor$ terms while the sets $\{\mathcal{I}_{\lfloor i/2 \rfloor, j'}(\mathcal{I}_{i,j}(\{1, \dots, N_r\}))\}_{j'=1}^{\binom{i}{\lfloor i/2 \rfloor}}$ (which correspond to all possible combinations of $\lfloor i/2 \rfloor$ elements of $\mathcal{I}_{i,j}(\{1, \dots, N_r\})$) span the entire set $\mathcal{I}_{i,j}(\{1, \dots, N_r\})$. Consequently, (54) can be written as:

$$P_{i,j}^{(N_r)} \doteq \min_{n_1, \dots, n_{\lfloor i/2 \rfloor} \in \mathcal{I}_{i,j}(\{1, \dots, N_r\})} \left\{ \beta_1^{(n_1)} + \dots + \beta_1^{(n_{\lfloor i/2 \rfloor})} \right\} \quad (55)$$

From (49) and (50), $\Pi_1 \doteq 0$ while $\Pi_2 \doteq \sum_{m_2 \in \{1, \dots, N_r\} \setminus \mathcal{I}_{i,j}(\{1, \dots, N_r\})} \beta_2^{(m_2)}$ where this summation contains $N_r - i$ summands. Consequently, from (50) and (55), the diversity order of scheme 1 can be written as:

$$P_{e|I}^{(N_r)} \doteq d_1 \triangleq \beta_0 + \min_{i=0, \dots, N_r} \{d(i)\} \quad (56)$$

$$= \beta_0 + \min_{i=0, \dots, N_r} \left\{ \min_{j=1, \dots, \binom{N_r}{i}} \left\{ \min_{n_1, \dots, n_{\lfloor i/2 \rfloor} \in \mathcal{A}_{i,j}} \left\{ \beta_1^{(n_1)} + \dots + \beta_1^{(n_{\lfloor i/2 \rfloor})} \right\} + \sum_{n \in \bar{\mathcal{A}}_{i,j}} \beta_2^{(n)} \right\} \right\} \quad (57)$$

where $\mathcal{A}_{i,j} \triangleq \mathcal{I}_{i,j}(\{1, \dots, N_r\})$ and $\bar{\mathcal{A}}_{i,j} \triangleq \{1, \dots, N_r\} \setminus \mathcal{I}_{i,j}(\{1, \dots, N_r\})$. In particular, $d(0) = \sum_{n=1}^{N_r} \beta_2^{(n)}$ and $d(N_r) = \min_{n_1, \dots, n_{\lfloor N_r/2 \rfloor} \in \{1, \dots, N_r\}} \left\{ \beta_1^{(n_1)} + \dots + \beta_1^{(n_{\lfloor N_r/2 \rfloor})} \right\}$.

We next compare the values of $d(2k-1)$ and $d(2k)$. Since $\lceil \frac{2k-1}{2} \rceil = \lceil \frac{2k}{2} \rceil = k$, then the summation $\beta_1^{(n_1)} + \dots + \beta_1^{(n_{\lceil i/2 \rceil})}$ in (57) contains the same number of summands for $i = 2k-1$ and $i = 2k$. Moreover, since the sets $\mathcal{A}_{2k,j}$ contains more

elements than $\mathcal{A}_{2k-1,j}$ (over the range of values of j), then the minimization over the former sets will result in a smaller minimum. Consequently:

$$\min_{n_1, \dots, n_k \in \mathcal{A}_{2k,j}} \left\{ \beta_1^{(n_1)} + \dots + \beta_1^{(n_k)} \right\} \leq \min_{n_1, \dots, n_k \in \mathcal{A}_{2k-1,j}} \left\{ \beta_1^{(n_1)} + \dots + \beta_1^{(n_k)} \right\} \quad (58)$$

In the same way, the summation $\sum_{n \in \bar{\mathcal{A}}_{i,j}} \beta_2^{(n)}$ in (57) contains $N_r - i$ terms. Consequently, over the entire range of values of j :

$$\sum_{n \in \bar{\mathcal{A}}_{2k,j}} \beta_2^{(n)} \leq \sum_{n \in \bar{\mathcal{A}}_{2k-1,j}} \beta_2^{(n)} \quad (59)$$

Consequently, $d(2k) \leq d(2k-1)$. As a conclusion, the minimization in (56) needs to be performed only over the even integers i in the set $\{1, \dots, N_r\}$. Moreover, the case $i = N_r$ needs to be included in the minimization even when N_r is odd. In fact, when $N_r = 2k-1$ is odd, the inequality $d(2k) \leq d(2k-1)$ will not imply the exclusion of $i = N_r$ from the minimization since $2k \notin \{1, \dots, N_r\}$. Finally, changing the minimization set in (57) results in (28).

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