A $2 \times 2$ Shape-Preserving ST Code for UWB Communications with Multipulse PPM

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Abstract—In this paper, we consider the application of Multipulse Pulse Position Modulation (MPPM) as a power-efficient modulation scheme for Impulse-Radio Ultra-Wideband (IR-UWB) communications and we propose the first known MPPM-specific $2 \times 2$ space-time code that is convenient for this modulation. The proposed rate-1 code achieves a full transmit diversity order with all MPPM constellations without introducing any constellation expansion. We also show that the proposed code profits from a low decoding complexity in the absence of interference between the different modulated UWB pulses.

Index Terms—Ultra-Wideband (UWB), Space-Time (ST) codes, Multipulse Pulse Position Modulation (MPPM).

I. INTRODUCTION

Unipolar transmissions are desirable for low-cost Ultra-Wideband (UWB) communication systems where it is complicated to control the amplitude and the phase of the very low duty-cycle sub-nanosecond modulated pulses. This can be achieved by deploying time-modulations such as Pulse Position Modulation (PPM) that takes advantage from the high temporal resolution of UWB systems [1]. An alternative time-modulation to PPM is Multipulse-Pulse Position Modulation (MPPM) that was considered in [2], [3] in the context of free-space optical communications. For $N$-multipulse-$M$-PPM, each symbol duration is divided into $M$ time slots and modulation is performed by transmitting $N < M$ pulses in $N$ distinct slots resulting in a signal constellation having a cardinality of $\binom{M}{N}$. While the interest in PPM resides in its energy efficiency, MPPM modulations are characterized by higher bandwidth efficiency and lower peak-to-average power ratios [2].

On the other hand, there is a growing interest in applying the Space-Time (ST) coding techniques on Impulse-Radio (IR) UWB systems [4], [5]. While the problem of ST coding for UWB systems was initially considered with conventional modulations such as BPSK, more recent contributions proposed new ST codes for PPM [6]–[9]. In what follows, we define a shape-preserving code as a code that does not introduce any expansion to the uncoded constellation; in other words, the transmitted coded symbols belong to the same signal set as the uncoded symbols. In this context, the PPM-ST codes that were proposed in the literature can be classified as non shape-preserving non-unipolar [6], non shape-preserving unipolar [7] (scheme 2) or shape-preserving such as the binary PPM code proposed in [8] and the $M$-ary PPM codes proposed in [7] (scheme 1) and [9].

In this paper, we consider the problem of ST coding with $N$-multipulse-$M$-PPM modulations and we propose the first known $2 \times 2$ code that can be applied with such constellations for all values of $M$ and $N$. In particular, the proposed code reduces to the $2 \times 2$ PPM-code proposed in [9] for the special case of $N = 1$. The proposed code is rate-1, fully diverse and shape-preserving with MPPM. Moreover, in the absence of Inter-Pulse-Interference (IPI), the proposed scheme can be associated with a low-complexity Maximum-Likelihood (ML) decoder whose complexity grows linearly with the dimensionality of the MPPM signal set. Finally, the number of pulses transmitted from the two antennas during consecutive symbol durations can be made different thus offering more scalability to the code and permitting to achieve different levels of compromise between data rate and error performance.

II. SYSTEM MODEL

Consider a Time-Hopping (TH) UWB system where the transmitter is equipped with $P = 2$ antennas. In what follows, we propose a minimal-delay diversity scheme that conveys two information symbols during two symbol durations. The multipulse modulation scheme is as follows: each symbol duration is divided into $M$ time slots, and for the communication of the first (resp. second) symbol, $N_1$ (resp. $N_2$) UWB pulses are transmitted in $N_1$ (resp. $N_2$) slots out of the $M$ possible slots. For this $M$-dimensional modulation, the $i$-th information symbol can be represented by the $M$-dimensional vector $s_i \triangleq [s_{i,1} \cdots s_{i,M}]^T$ for $i = 1, 2$ where exactly $N_i$ components among $s_{i,1}, \ldots, s_{i,M}$ are equal to 1 while the remaining components are equal to 0. In other words:

$$s_i \in C_i \triangleq \left\{ \sum_{k=1}^{N_i} e_{n_k} ; n_1 \neq \cdots \neq n_N \right\} ; \ i = 1, 2 \quad (1)$$

where $e_m$ stands for the $m$-th column of the $M \times M$ identity matrix $I_M$. Note that $M$-ary PPM follows as a special case by setting $N_1 = N_2 = 1$.

Assume that the receiver is equipped with $Q$ antennas and that each antenna is followed by an $L$-th order Rake that combines the first $L$ arriving multi-path components. At each finger of the Rake, a bank of $M$ correlators is used to separate the different components of the $M$-dimensional transmitted symbols. For a single-user scenario in the absence of inter-symbol-interference, the linear dependence between the baseband inputs and outputs of the channel can be expressed as:

$$Y = HC + N \quad (2)$$
where $C$ is the $2M \times 2$ codeword. The $(p-1)M + m, t)$-th entry of $C$ can be equal to either 0 or 1 indicating, respectively, the absence or presence of an UWB pulse at the $m$-th position of the $p$-th antenna during the $t$-th symbol duration for $p = 1, 2, m = 1, \ldots, M$ and $t = 1, 2$. The matrices $Y$ and $N$ are $QLM \times 2$ matrices corresponding to the decision variables and the noise terms respectively.

In (2), $H$ is the $QLM \times 2M$ channel matrix that can be written as: $H = [H^{(1)}\ H^{(2)}]$. $H^{(p)}$ is a $QLM \times M$ matrix whose $((q-1)L,M+(l-1)M+m, m')$-th element corresponds to the impact of the signal transmitted during the $m'$-th position of the $p$-th antenna on the $m$-th correlator (corresponding to the $m$-th PPM time slot) placed after the $l$-th Rake finger of the $q$-th receive antenna. This element is given by:

$$H^{(p)}((q-1)L,M + (l-1)M + m, m') = \int_{-\infty}^{+\infty} g_{q,p}(t)w(t - \Delta_l - (m-m'))\ dt \quad (3)$$

where $w(t)$ stands for the pulse waveform having a duration $T_w$ and $g_{q,p}(t)$ stands for the convolution of $w(t)$ with the impulse response of the frequency-selective channel between antennas $p$ and $q$. In (3), $\delta$ stands for the duration of each PPM time slot ($\delta \geq T_w$) and $\Delta_l \equiv (l-1)T_w$ stands for the delay of the $l$-th finger of the Rake. Interested readers are referred to [9] for more details on the system model.

### III. Code Construction

In this paper, we propose the following structure for the $(2M \times 2)$-dimensional codewords:

$$C(s_1, s_2) = \left[ \begin{array}{c} s_1 \\ \Omega s_2 \\ s_1 \end{array} \right] ; \ s_1 \in C_1 ; \ s_2 \in C_2 \quad (4)$$

where $C_1$ and $C_2$ are the $M$-dimensional signal sets given in (1). $\Omega$ is the $M \times M$ cyclic permutation matrix given by:

$$\Omega = \left[ \begin{array}{c} 0_{1 \times (M-1)} \\ I_{M-1} \\ 0_{(M-1) \times 1} \end{array} \right] \quad (5)$$

where $0_{m \times n}$ stands for the $m \times n$ matrix whose components are all equal to zero.

Evidently, $\Omega s_2 \in C_2$ whenever $s_2 \in C_2$ implying that the proposed code does not result in any expansion of the multipulse constellations. In other words, the transmitted ST-coded symbols belong to either $C_1$ or $C_2$ (just like the uncoded symbols) implying that the proposed code is shape-preserving with all multipulse constellations.

**Proposition**: The proposed code achieves a full transmit diversity order for all values of $M$, $N_1$, and $N_2$. In particular, the proposed code is fully diverse in the case where $N_1 = N_2$ (i.e. when both information symbols are carved from the same multipulse signal set).

**Proof**: Denote by $A_i$ the set of all possible differences between two information vectors carried from $C_i$ for $i = 1, 2$:

$$A_1 \equiv \{s-s'; \ s, s' \in C_1\} ; \ A_2 \equiv \{s-s'; \ s, s' \in C_2\} \quad (6)$$

Based on the rank criterion [10], the proposed code is fully diverse if the matrix $C(a_1, a_2)$ has a full rank for $(a_1, a_2) \in A_1 \times A_2\{(0_M, 0_M)\}$ where $0_M$ stands for the $M$-dimensional all-zero vector. From (4) and (5), $C(a_1, a_2)$ can be written as:

$$C(a_1, a_2) = \left[ \begin{array}{cccc} a_{1,1} & \cdots & a_{1,M} & a_{2,1} & \cdots & a_{2,M} \\ a_{2,1} & \cdots & a_{2,M} & a_{1,1} & \cdots & a_{1,M} \end{array} \right] \quad (7)$$

where $\pi(.)$ stands for the cyclic permutation of order:

$$\pi(i) = (i \rightarrow i-2) \mod M + 1 \quad (8)$$

In what follows, $C(a_1, a_2)$ will be denoted by $C$ when there is no ambiguity. On the other hand, rank($C) < 2$ if there exists a nonzero real number $k$ such that $C_2 = kC_1$ where $C_i$ stands for the $i$-th column of $C$ for $i = 1, 2$. Let $m$ be an integer in the set $\{1, \ldots, M\}$. Investigating the $m$-th and $(M + m)$-th rows of $C$ respectively, the relation $C_2 = kC_1$ implies that:

$$a_{2,m} = ka_{1,m} ; \ m = 1, \ldots, M \quad (9)$$

$$a_{1,m} = ka_{2,\pi(m)} ; \ m = 1, \ldots, M \quad (10)$$

Combining the above equations results in:

$$a_{1,m} = k^2a_{1,\pi(m)} ; \ m = 1, \ldots, M \quad (11)$$

The above equation implies that if $C$ is rank-deficient and if one component among $a_{1,1}, \ldots, a_{1,M}$ is equal to zero, then the remaining components will be all equal to zero since the function $\pi(.)$ defines a circular permutation over the set $\{1, \ldots, M\}$.

Assume that $N_1 < \frac{M}{2}$: Since exactly $N_1$ components of elements of $C_1$ are different from zero (and equal to one), then at most $2N_1$ components of elements of $A_1$ can be different from zero. Since in this case $2N_1 < M$, then there is at least one component of $a_1$ that is equal to zero resulting in $a_{1,1} = \cdots = a_{1,M} = 0$ following from (11). Replacing these values in (9) results in $a_{2,1} = \cdots = a_{2,M} = 0$. Therefore, if $N_1 < \frac{M}{2}$ then the only rank-deficient matrix is the all-zero code $C(0_M, 0_M)$.

Assume now that $N_1 > \frac{M}{2}$: Vector $a_1$ can be written as: $a_1 = s_1 - s'$ where $s_1, s' \in C_1$. Since $N_1$ components of $s_1$ and $N_1$ components of $s'$ are equal to one, then the relation $2N_1 > M$ will imply that there is at least one integer $i \in \{1, \ldots, M\}$ such that $s_{1,i} = s'_{1,i} = 1$. In this case, $a_{1,i} = 0$ resulting in $a_{1,1} = \cdots = a_{1,M} = 0$ and $a_{2,1} = \cdots = a_{2,M} = 0$ following from equations (11) and (9), respectively. Therefore, if rank($C(a_1, a_2)$) < 2 and $N_1 > \frac{M}{2}$, then $a_1 = a_2 = 0_M$.

If $M$ is odd, then $N_1$ is either strictly less than $\frac{M}{2}$ or strictly greater than $\frac{M}{2}$. Consequently, based on the previous discussion, if rank($C(a_1, a_2)$) < 2 and $M$ is odd, then $a_1 = a_2 = 0_M$ implying that the only rank-deficient matrix is the all-zero code $C(0_M, 0_M)$. Therefore, the code is fully diverse for all values of $N_1$ when $M$ is an odd integer.

Assume now that $M$ is even. For a given value of $N_1$, if one of the relations $N_1 < \frac{M}{2}$ or $N_1 > \frac{M}{2}$ is satisfied, then as before, the only rank-deficient matrix will be the all-zero matrix. Now consider the case $N_1 = \frac{M}{2}$ (which can be satisfied since $M$ is even). If at least one component of $a_1$ is equal to 0, then from what preceded rank($C(a_1, a_2)$) < 2 if and only if $a_1 = a_2 = 0_M$. If all $M = 2N_1$ components of $a_1$ are different from zero then, following from the structure of the set $A_1$ given in (6), exactly $N_1$ components of $a_1$ must be equal to +1 while the remaining $N_1$ components must be
all equal to $-1$. Consequently, (11) cannot be satisfied in this case. In fact, since $k^2 > 0$, then (11) holds if and only if all components of $a_1$ have the same sign. Since (11) does not hold then rank$(C(a_1, a_2)) = 2$. Therefore, the only rank-deficient matrix is the all-zero matrix for the cases $N_1 < \frac{M}{2}$, $N_1 > \frac{M}{2}$ or $N_1 = \frac{M}{2}$; that is, for all values of $N_1$.

Note that since the above proof holds independently from the value taken by $N_2$, then the proposed code is fully diverse for all values of $M$, $N_1$ and $N_2$. In particular, if it is desired that all the transmitted symbols must belong to the same $M$-dimensional $N$-multipulse signal set, we can set $N_1 = N_2 = N$ without modifying the diversity order of the proposed scheme. Note that the $2 \times 2$ PPM-code proposed in [9] follows as a special case by setting $N_1 = N_2 = 1$.

Finally, it is worth noting that the transmit diversity is achieved because of the particular structure of the sets $A_1$ and $A_2$ given in (6). For example, the matrix $C(a_1, a_2)$ does not have a full rank when $a_1$ and $a_2$ have all their components equal to 1. However, these vectors simply do not belong to the sets $A_1$ and $A_2$ for all values of $M$, $N_1$ and $N_2$.

IV. ML DECODER

A. Decoder Structure

Denote by $T_c$ the maximum delay spread of the UWB channel ($T_c \gg T_w$). The Inter-Pulse-Interference (IPI) between the different PPM slots can be eliminated by setting $\delta \geq T_c + T_w$.

We next present a simple ML decoder that can be applied in the different PPM slots. In fact, since $k_1^2$ case. In the absence of IPI, the channel coefficients in (3) can be removed from the metric. On the other hand, following the structure of the multipulse constellation, $\sum_m a_{1,m} = \sum_m s_{1,m} = N_1$ for all values of $s_1$ and $\sum_m s_{2,m} = N_2$ for all values of $s_2$ implying that $d_{s_1,s_2}$ and $d_{s_2,s_1}$ can be ignored when determining $d_{s_1,s_2}$. In the same way, since $\pi(.)$ defines a bijection over the set $\{1, \ldots, M\}$, then $\sum_m s_{2,\pi(m)} = N_2$ implying that $d_{s_1,s_2}$ can be removed from the summation. Replacing $m$ by $\pi(m)$ in $d_{s_1,s_2}$ implies that this term can be written as: $-2 \sum_m s_{2,m} \sum_{q,l} h_{q,l}^{(1)} y_{q,l,m} \prod_{i=1}^{M+1} \pi^{-1}(m)$ where $\pi^{-1}(i) = i \mod M+1$ stands for the inverse permutation.

Based on the above simplifications, the metric $d_{s_1,s_2}$ is equivalent to:

$$d_{s_1,s_2} = - \sum_m k_{1,m} s_{1,m} - \sum_m k_{2,m} s_{2,m} + k_3 \sum_m s_{1,m} \left( s_{2,m} + s_{2,\pi(m)} \right)$$

(15)

where:

$$k_{1,m} = \sum_{q,l} h_{q,l}^{(1)} y_{q,l,m}$$

(16)

$$k_{2,m} = \sum_{q,l} h_{q,l}^{(1)} y_{q,l,m}$$

(17)

$$k_3 = \sum_{q,l} h_{q,l}^{(1)} y_{q,l,m}$$

(18)

(19)

(15)

(16)

(17)

(18)

(19)

(15)

(16)

(17)

(18)

(19)

(15)

(16)

(17)

(18)

(19)
Finally, based on (15), the ML decoder decides in favor of 
\( \hat{s}_1 \) and \( \hat{s}_2 \) such that:

\[
(\hat{s}_1, \hat{s}_2) = \arg \max_{s_2 \in C_2} \left[ -d_{s_1,s_2} \right]
\]

\[
= \arg \max_{s_2 \in C_2} \left[ \sum_{m \in S_1|s_2} k_1, m + \sum_{m \in S_2} k_2, m - k_3 \sum_{m \in S_1|s_2} (s_{2,m} + s_{2,\pi(m)}) \right]
\]

where the set \( C_2 \) given in (1) contains \( \binom{M}{N_2} \) elements and the set \( S_1|s_2 \) (containing \( N_1 \) elements) is determined from \( s_2 \) according to (19).

B. Decoder Complexity

Note that, unlike the direct implementation of ML decoders, (20) avoids evaluating the \( \binom{M}{N_1} \binom{M}{N_2} \) metrics associated with all elements \((s_1, s_2) \in C_1 \times C_2\). This was realized by determining \( s_1 \) from \( s_2 \) in a unique way according to (19). In what follows, we evaluate the complexity of the proposed decoder in terms of the number of multiplications required for decoding a pair of information symbols \((s_1, s_2)\). This simplified approach neglects the complexity of the additions and comparisons involved in the decoding algorithm.

The ML decoding procedure is as follows:

1) The receiver calculates the \( 2M + 1 \) constants \( \{k_1, m\}_{m=1}^{M}, \{k_2, m\}_{m=1}^{M} \) and \( k_3 \) given in (16)-(18). This step necessitates \( 2MQL + 2MQL + QL = (4M+1)QL \) multiplications.

2) For each value of \( s_2 \) (among \( \binom{M}{N_2} \) possible values), the decoder determines the value of \( s_1 \) according to (19). There are no multiplications involved in this procedure. In fact, given that the term \( s_{2,m} + s_{2,\pi(m)} \) can be equal to either 0, 1 or 2, then the term \( k_1, m - k_3 (s_{2,m} + s_{2,\pi(m)}) \) can be calculated by performing 0, 1 or 2 additions respectively without necessitating any number of multiplications. In the same way, the sorting function \( \text{MAX}[\cdot] \) can be implemented based on comparators and, hence, does not require any number of multiplications.

3) Once the \( \binom{M}{N_2} \) candidate symbol pairs \((s_1|s_2, s_2)\) are determined, their corresponding metrics \( d_{s_1|s_2, s_2} \) are calculated and the decision is made according to (20). Once again, since \((s_{2,m} + s_{2,\pi(m)}) \in \{0, 1, 2\}\), then there are no multiplications involved in this step.

From what preceded, the ML decoder requires \((4M+1)QL\) multiplications for the detection of one pair of information symbols. Note that this complexity increases linearly with the dimensionality \( M \) of the signal set and it does not depend on values taken by \( N_1 \) and \( N_2 \). Note also that since there are no up-converters and down-converters in IR-UWB systems, the channel coefficients (as well as the modulated symbols) are all real-valued; consequently, all of the above multiplications correspond to real-valued multiplications. Even though the proposed code is not orthogonal, its complexity is comparable to that of the orthogonal Alamouti code [12]. In fact, for decoding one pair of one-dimensional PAM symbols, the Alamouti code requires \( 4QL \) real-valued multiplications. In other words, the additional complexity of the proposed decoder

(in the order of \( \frac{4M+1}{4} \approx M \) for large values of \( M \)) follows mainly from the dimensionality of the signal set and not from the structure of the encoder/decoder.

V. SIMULATIONS AND RESULTS

Simulations are performed over the IEEE 802.15.3a channel model recommendation CM2 [13]. A Gaussian pulse of duration \( T_w = 0.5 \) ns is used. In the no-IPI case, the modulation delay is chosen as \( \delta = 100 \) ns which is larger than the maximum delay spread of the UWB channel while in the IPI case we set \( \delta = 0.5 \) ns. The presented results show the variation of the Symbol-Error-Rate (SER) with the Signal-to-Noise Ratio (SNR) per bit. The proposed code transmits at the rate of \( \frac{\log_2 \left( \binom{N}{1}\binom{N}{1} \right)}{2} \) bits per channel use (bpu) while \( N \)-multipulse-\( M \)-PPM transmits at the rate of \( \log_2 \left( \binom{N}{1} \right) \) bpu. Given that the proposed \( 2 \times 2 \) (resp. \( 1 \times 1 \)) system transmits on the average \( N_1+N_2 \) (resp. \( N \)) pulses per symbol, then the energy of the UWB Gaussian pulse must be normalized by \( \frac{1}{\sqrt{(N_1+N_2)/2}} \) (resp. \( \frac{1}{N} \)).

Figures 1 and 2 compare the performance of the proposed code with that of single-antenna systems in the absence and presence of IPI, respectively. Results show the high performance levels and the enhanced diversity order achieved by the proposed scheme. In coherence with [3], under a fixed average power constraint, multipulse systems permit to achieve higher bit rates at the expense of deteriorating the performance. \( N \)-multipulse-\( M \)-PPM ST-coded IR-UWB systems can constitute a strong candidate solution for high rate wireless communications; in particular, this modulation scheme is more bandwidth-efficient compared to \( M \)-PPM. For example, Fig. 1 shows that the ST-coded multipulse system with \( (M,N) = (8,3) \) outperforms the single-antenna 8-ary PPM system by approximately 3.5 dB (at a SER of \( 10^{-3} \)) while transmitting 1.93 times faster. Finally, based on the decoding procedure described in (19)-(20), it is more convenient to choose \( N_2 \leq N_1 \). For example, in Fig. 2, the proposed scheme with \((N_1,N_2) = (1,2)\) would achieve the same error performance and data rate as the \((N_1,N_2) = (2,1)\) system.
but at the expense of a slightly higher decoding complexity (more additions and comparisons but the same number of multiplications).

VI. CONCLUSION

We introduced a new construction method for unipolar shape-preserving MPPM ST codes with two transmit antennas for any signal-set dimensionality. This novel construction that was based on pulse permutations can open the door to new families of codes that can be applied with any number of transmit antennas.

REFERENCES


