A 2×2 Antennas Ultra-Wideband System with Biorthogonal Pulse Position Modulation

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***Abstract*— The Golden code [1] is a 2×2 space-time code that achieves the best known performance with all constellations carved from $\mathbb{Z}[i]$. In this letter, we present the construction of a new coding scheme for $2M$-ary biorthogonal pulse position modulations (BPPM) with $M \geq 4$. The proposed code satisfies all of the construction constraints of the Golden code and it has the additional advantage of being totally real making it suitable for low cost carrier-less ultra-wideband terminals. Namely, this totally real construction achieves full rate and full diversity with the best known coding gain and without any shaping losses for $2M$-BPPM with $M \geq 4$.**

***Index Terms*— UWB, Space-Time, MIMO, BPPM, PPM.**

I. Introduction

RECENTLY UWB WPANs (IEEE 802.15.3) have drawn considerable attention for short range radio links. For these systems the BPPM signals achieve high data rates with a good compromise between complexity and performance [2]. Combining UWB with multi-antenna techniques can increase the spectral efficiency and reduce the error rate [3], [4].

All of the known space-time (ST) coding schemes were constructed over the hypercubes carved from the lattice of rational integers [1], [5]. The reason is that this generic construction keeps its properties when associated with the most popular modulation schemes which are subsets of these hypercubes (ex. QAM, PAM, PPM, ...). Instead of adopting this approach, we exploited the structure of the BPPM constellations in order to construct a BPPM-specific code. This code keeps the natural advantages of carrier-less impulse radio BPPM-UWB (no need for frequency synthesizers, low cost, ...) and achieves better performance with higher spectral efficiency.

Notations: I_n is the $n \times n$ identity matrix. O_n and 1_n correspond to the n-dimensional vectors whose elements are equal to 0 and 1 respectively. $\mathrm{vec}(X)$ stacks the columns of the matrix X vertically. $\otimes$ corresponds to the Kronecker product.

II. System Model

In hybrid M-PPM and M'-PAM, the input data is modulated onto both the pulse amplitudes and pulse positions. Each element of this constellation is represented by an M-dimensional vector that belongs to the set:

$$\mathcal{C} = \{(2m'-1-M')e_{m+1}; m'=1,...,M'; m=0,...,M-1\}$$

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where e_m is the m-th column of I_M. $2M$-BPPM is a special case obtained by setting $M'=2$. In single-user time hopping UWB, the signal transmitted from the p-th antenna is:

$$s_p(t) = \sum_{n=0}^{N_f-1}\sum_{m=0}^{M-1} a_{p,m} w(t - nT_f - m\delta) \qquad (1)$$

where $w(t)$ is the monocycle pulse waveform of duration T_w normalized to have unit energy. N_f pulses are used to convey each information symbol. Each one of these pulses is emitted during one time frame of duration T_f. δ is the modulation delay and is chosen to satisfy $\delta \geq T_w$. $a_p = [a_{p,0}, ..., a_{p,M-1}]^T \in \mathcal{C}$ is composed of $M-1$ zero values and one component that belongs to the M'-ary PAM constellation.

The received signal at the q-th antenna is given by:

$$r_q(t) = \sum_{p=1}^{P}\sum_{n=0}^{N_f-1}\sum_{m=0}^{M-1} a_{p,m} h_{q,p}(t - nT_f - m\delta) + n_q(t) \qquad (2)$$

where $n_q(t)$ is the noise at the q-th antenna which is supposed to be real AWGN with double sided spectral density $PN_0/2$. $h_{q,p}(t)$ is the convolution of $w(t)$ and $g_{q,p}(t)$ which stands for the impulse response of the frequency selective channel between the p-th transmit and the q-th receive antenna.

In order to take advantage of the multi-path diversity, an L-th order Rake receiver is used. The finger delays are chosen as $\Delta_l = lMT_w$ for $l = 0, ..., L-1$. This corresponds to combining the first arriving multi-path components. In the absence of inter symbol interference, the QLM decision variables take the form:

$$\begin{aligned} z_{q,l,m} &= \int_0^{N_fT_f} r_q(t) \sum_{n=0}^{N_f-1} w(t - nT_f - \Delta_l - m\delta)\mathrm{d}t \\ &= \sum_{p',m'} a_{p',m'} r_{q,p'}((m-m')\delta + \Delta_l) + n_{q,l,m} \qquad (3) \end{aligned}$$

where: $r_{q,p}(\tau) = \int_0^{T_f} h_{q,p}(t) w(t-\tau)\mathrm{d}t$. $n_{q,l,m}$ is a white Gaussian noise which follows from $\delta \geq T_w$ and $\Delta_l = lMT_w$.

The last equation can be expressed in matrix form as:

$$Z = RC + N \qquad (4)$$

Z and N are the $QLM \times T$ decision and noise matrices respectively. For ST codes with temporal extension of T, the t-th column of the $PM \times T$ matrix C is the vertical concatenation of $a_1, ..., a_P$. R is the channel matrix of dimensions $QLM \times PM$: $R = [R_1^T, \dots, R_Q^T]^T$. $R_q = [R_{q,0}^T, \dots, R_{q,L-1}^T]^T$ is a $LM \times PM$ matrix corresponding to the q-th receive antenna. The $M \times PM$ constituent matrices $R_{q,l}$ take the form $R_{q,l} = [R_{q,l,1} \ \cdots \ R_{q,l,P}]$. $R_{q,l,p}$ is a $M \times M$ matrix whose (m,m')-th element is given by: $R_{q,l,p}(m,m') = r_{q,p}(\Delta_l + (m-m')\delta)$.

III. Code Construction

Consider the quadratic field extension given by:

$$\mathbb{K} = \mathbb{Q}(\sqrt{5}) = \{a + b\sqrt{5} \mid a, b \in \mathbb{Q}\} \quad (5)$$

For 2 transmit antennas and M-dimensional constellations, the coding scheme is constructed over the ring of integers of $\mathbb{K}$ given by $\mathcal{O}_{\mathbb{K}} = \mathbb{Z}(\theta) = \{a + b\theta \mid a, b \in \mathbb{Z}\}$ with $\theta = \frac{1+\sqrt{5}}{2}$. Each codeword is given by the $2M \times 2$ matrix:

$$C = \text{diag}(\sqrt{\alpha} 1_M^T \ \ \sqrt{\sigma(\alpha)} 1_M^T) \begin{bmatrix} a_1 + \theta a_2 & a_3 + \theta a_4 \\ \Omega(a_3 + \bar{\theta} a_4) & a_1 + \bar{\theta} a_2 \end{bmatrix} \quad (6)$$

where $a_i \in \mathcal{C}$ are the M-dimensional vector representations of the transmitted symbols for $i = 1, ..., 4$. $\bar{\theta} = \sigma(\theta) = -\frac{1}{\theta}$ is the conjugate of θ. Ω is the $M \times M$ permutation matrix given by:

$$\Omega = \begin{bmatrix} \mathrm{O}_{M-1}^T & 1 \\ I_{M-1} & \mathrm{O}_{M-1} \end{bmatrix} \quad (7)$$

The multiplication by the diagonal matrix in eq. (6) corresponds to normalizing the transmitted energy. $\alpha = \frac{3-\theta}{5}$ and $\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(\sqrt{\alpha}) = \sqrt{\alpha\sigma(\alpha)} = \frac{1}{\sqrt{d_{\mathbb{K}}}}$ where $d_{\mathbb{K}} = 5$ is the discriminant of $\mathbb{K}$. This corresponds to limiting the construction in the ideal $I = \sqrt{\alpha}\mathcal{O}\mathbb{K}$ whose volume is equal to 1 (please refer to [4] for more details).

In what follows, we will show that this coding scheme is fully diverse for combined M-PPM-M'-PAM constellations with $M > 2$ and for all values of M'. $2M$-BPPM and M-PPM follow as special cases by setting $M' = 2$ and $M' = 1$ respectively. Ignoring the normalization matrix in eq. (6) which is common to all codewords, let

$$\Delta C(X, Y) = C - C' = \begin{bmatrix} X & Y \\ \Omega\sigma(Y) & \sigma(X) \end{bmatrix} \quad (8)$$

We must show that the rank of $\Delta C(X, Y)$ is equal to 2 for all values of $(X, Y) \neq (\mathrm{O}_M, \mathrm{O}_M)$ where $X, Y \in \mathcal{A}$:

$$\mathcal{A} = \{(a - a') + (b - b')\theta \ \mid \ a, a', b, b' \in \mathcal{C}\} \subset \mathcal{O}_{\mathbb{K}}^M \quad (9)$$

Proposition 1: if $\exists i | X_i = 0$ then $\text{rank}(\Delta C(X, Y)) = 2$ unless $X = Y = \mathrm{O}_M$.

Proof: Designate by π the cyclic permutation given by: $\pi(i) = i \mod M + 1$ for $i = 1, ..., M$.

$$\Delta C = \begin{bmatrix} X_1 & \cdots & X_M & \sigma(Y_{\pi^{-1}(1)}) & \cdots & \sigma(Y_{\pi^{-1}(M)}) \\ Y_1 & \cdots & Y_M & \sigma(X_1) & \cdots & \sigma(X_M) \end{bmatrix}^T \quad (10)$$

Suppose that $\text{rank}(\Delta C) < 2$ then its two columns have the same direction. Therefore, considering the first M rows of ΔC, $X_i = 0 \Rightarrow Y_i = 0$. Now we have $\sigma(Y_i) = 0$ (since $Y_i = 0$ and $\{1, \theta\}$ is an integral basis of $\mathcal{O}_{\mathbb{K}}$). Considering the last M rows of ΔC, $\sigma(Y_i) = 0 \Rightarrow \sigma(X_{\pi(i)}) = 0 \Rightarrow X_{\pi(i)} = 0$. Starting the same procedure again with $\pi(i)$ rather than i, we can conclude by iteration that $X_i = X_{\pi(i)} ... = X_{\pi^{M-1}(i)}$ and $Y_i = Y_{\pi(i)} ... = Y_{\pi^{M-1}(i)} \Leftrightarrow X = Y = \mathrm{O}_M$ since π is a bijection over $\{1, ..., M\}$. The same proof holds if $\exists i | Y_i = 0$.

Lemma 1: The code achieves full diversity for $M > 4$.

Proof: From the definition of $\mathcal{A}$ in eq. (9), the vectors X and Y are linear combinations of any 4 columns of I_M. Therefore for $M > 4$, X and Y each have at least one zero component resulting in full rank as shown in proposition 1.

We must now verify that ΔC has a full rank when all of its components are nonzero. When ΔC has no zero components, $\text{rank}(\Delta C) < 2$ implies that:

$$\frac{Y_1}{X_1} = \ldots = \frac{Y_M}{X_M} = \frac{\sigma(X_1)}{\sigma(Y_{\pi^{-1}(1)})} = \ldots = \frac{\sigma(X_M)}{\sigma(Y_{\pi^{-1}(M)})} = k \quad (11)$$

where $k \in \mathbb{K}$. After some manipulations, eq. (11) becomes:

$$X_1 = (\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(k))^{M+1-i} X_i \ \ ; \ \ i = M, M-1, ...2 \quad (12)$$

On the other hand, $X_i, Y_i \in \mathcal{O}_{\mathbb{K}}^* = \mathbb{Z}^* \oplus \theta\mathbb{Z}^* \oplus \mathcal{O}'_{\mathbb{K}}$ for all values of i where $\mathcal{O}'_{\mathbb{K}} = \{a + b\theta \mid a, b \in \mathbb{Z}^* = \mathbb{Z} - \{0\}\}$. Since $\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(k) \in \mathbb{Q}$ for $k \in \mathbb{K}$, eq. (12) implies that $X_1, ..., X_M$ (and $Y_1, ..., Y_M$ in an equivalent manner) must belong to one of the following sets $\mathbb{Z}^*$, $\theta\mathbb{Z}^*$ or $\mathcal{O}'_{\mathbb{K}}$ simultaneously.

Following from the structure of $\mathcal{A}$, a maximum number of 2 components of X can contain an integer or an integral multiple of θ. Therefore the code is nut fully diverse with $M = 2$. From eq. (9), both entries of X can belong to $\mathbb{Z}^*$ (resp. $\theta\mathbb{Z}^*$) when $b = b'$ (resp. $a = a'$) and $(a, a') = (x_1 e_i, x_2 e_j)$ (resp. $(b, b') = (x_1 e_i, x_2 e_j)$) for $i, j = 1, 2$ and $i \neq j$. In the same way, X_1 and X_2 can both be in $\mathcal{O}'_{\mathbb{K}}$. In this case, the vector X takes the form $X = (x_1 + \theta x_2)e_i + (x_3 + \theta x_4)e_j$ where $x_1, ..., x_4$ are symbols of the M'-ary PAM and $i \neq j$.

For $M = 3$, when $X_i \neq 0$ for $i = 1, ..., M$, X belongs to the set of all possible permutations of:

$$\mathcal{A}' = \{[x_1, x_2, x_3\theta]^T, [x_1, x_2\theta, x_3\theta]^T, [x_1, x_2\theta, x_3 + x_4\theta]^T\}$$

where $x_1, ..., x_4 \in \mathbb{Z}^*$. Therefore, a maximum number of 2 components of X can be in $\mathbb{Z}^*$ (or $\theta\mathbb{Z}^*$) at the same time while only one component can belong to $\mathcal{O}'_{\mathbb{K}}$. This is in contradiction with eq. (12) which proves that the proposed code is fully diverse.

For $M = 4$, eq. (12) is in contradiction with the structure of $\mathcal{A}$. When $X_i \neq 0$ for $i = 1, ..., M$, and in order to occupy 4 positions, X must be a permutation of the vector $x_1 e_1 + \theta x_2 e_2 + x_3 e_3 + \theta x_4 e_4$. This implies that there are two values $X_i, X_j \in \mathbb{Z}^*$ while the other 2 values $X_k, X_l \in \theta\mathbb{Z}^*$. Therefore, the components of X can not belong simultaneously to $\mathbb{Z}^*$, $\theta\mathbb{Z}^*$ or $\mathcal{O}'_{\mathbb{K}}$. As a conclusion, C achieves full diversity for $M > 2$ and for all values of M'.

Proposition 2: C is information lossless.

Proof: Equation (4) can be expressed as:

$$\text{vec}(Z) = (I_2 \otimes R)\Phi[a_1^T \ a_2^T \ a_3^T \ a_4^T]^T + \text{vec}(N) \quad (13)$$

where, from eq. (6), $\text{vec}(C) = \Phi[a_1^T \ a_2^T \ a_3^T \ a_4^T]^T$ and:

$$\Phi = \begin{pmatrix} \sqrt{\alpha} I_M & \sqrt{\alpha}\theta I_M & \Theta_M & \Theta_M \\ \Theta_M & \Theta_M & \sqrt{\sigma(\alpha)}\Omega & \sqrt{\sigma(\alpha)}\bar{\theta}\Omega \\ \Theta_M & \Theta_M & \sqrt{\alpha} I_M & \sqrt{\alpha}\theta I_M \\ \sqrt{\sigma(\alpha)} I_M & \sqrt{\sigma(\alpha)}\bar{\theta} I_M & \Theta_M & \Theta_M \end{pmatrix} \quad (14)$$

where Θ_M is the $M \times M$ all-zero matrix. We can prove that $\Phi\Phi^T = I_{4M}$ which follows from the fact that the basis $\sqrt{\alpha}\{1, \theta\}$ is orthonormal and $\Omega\Omega^T = I_M$. The fact that Φ is unitary is sufficient for C to be information lossless [6].

When associated with M-dimensional constellations, the Golden code (denoted as G) can be obtained from eq. (6) by replacing Ω with $i = \sqrt{-1}$ and $\sqrt{\alpha}$ by $\alpha = 1 + i(1 - \theta)$

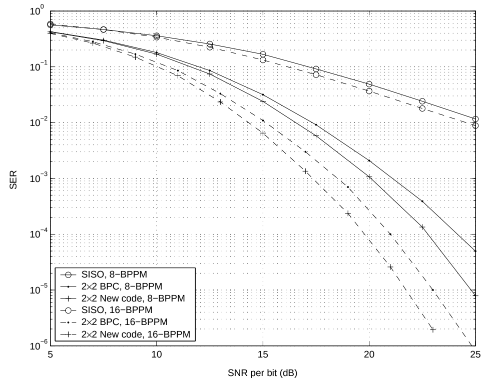

Fig. 1. The proposed code vs. the best previously known totally-real code (BPC) [4] with $L = 1$.

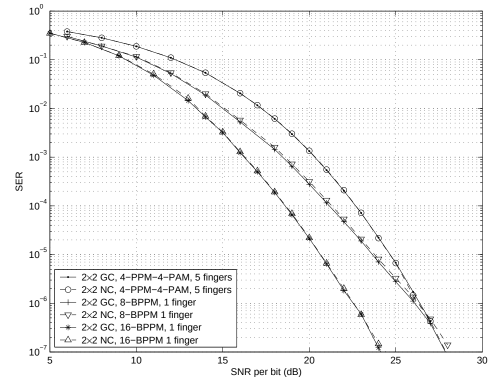

Fig. 2. The new code (NC) vs. the Golden Code (GC) [1].

[1]. When associated with finite QAM or PAM constellations, this code achieves the following coding gain [1]:

$$\delta_{min}(G) = \min_{X,Y \in \mathcal{A}, X \neq Y} |\det \Delta G(X,Y)| = \frac{4}{\sqrt{5}} \tag{15}$$

For M-dimensional constellations with $M \geq 2$, by rearranging the rows of $\Delta G = G - G'$, it can be written as:

$$\Delta G = [\Delta G_1^T, \ldots, \Delta G_r^T, \ldots, \Delta G_M^T]^T \tag{16}$$

where ΔG_m comprises the m-th and the $(M+m)$-th rows of ΔG for $m = 1, ..., M$ and r designates the number of such nonzero matrices. $r = 1$ implies that the antennas are transmitting at the same position during consecutive symbol durations and it follows that the coding gain is the same as in eq. (15). For $r > 1$: $\det(\Delta G^T \Delta G) \geq r^2 \min_{m=1 \cdots r}\{\det(\Delta G_m^T \Delta G_m)\}$. For a given value of r, the minimum nonzero value of the above equation is $r^2 \delta_{min}(G)^2/2^4$ since the difference between 2 data symbols now belongs to $\{m'\}^M$ rather than $\{2m'\}$ as in the case of PAM symbols for $m' \in [-M' \ M']$. So the minimal nonzero value of the coding gain is obtained for $r = 2$:

$$\delta_{min}(G_{M-\text{PPM}-M'-\text{PAM}}) = \frac{\delta_{min}(G)}{2} = \frac{2}{\sqrt{5}} \tag{17}$$

In other words, if the vector $b = [a_1 - a_1', ..., a_4 - a_4']$ is one of the vectors that minimizes $\delta_{min}(G)$ over the PAM constellation, then the vectors $(a_i e_p \pm a_i' e_{p'})$ for $i = 1, .., 4$ and $p \neq p'$ will yield half this minimum over the extended M-dimensional constellation for $p, p' = 1, ..., M$.

By numerical evaluation, we find that the coding gain of the proposed code when associated with $2M$-BPPM constellations is $\delta_{min}(C_{6-\text{BPPM}}) = \sqrt{1.553}/\sqrt{5}$ and $\delta_{min}(C_{2M-\text{BPPM}}) = 2/\sqrt{5}$ for $M = 4, ..., 8$. Moreover, $\delta_{min}(C_{2M'-\text{BPPM}}) \geq \delta_{min}(C_{2M-\text{BPPM}})$ for $M' \geq M$ since $2M'$-BPPM is obtained by adding new dimensions to $2M$-BPPM for $M' \geq M$. When associated with M-PPM, the coding gain is $\delta_{min}(C_{M-\text{PPM}}) = 2/\sqrt{5}$ for $M \geq 3$. This shows that the proposed scheme achieves the same coding gain as the Golden code for $2M$-BPPM with $M \geq 4$ and for M-PPM with $M \geq 3$. Since $2M$-BPPM has better performance and higher rate than M-PPM with the same complexity [2], it is more interesting to use UWB transceivers merging the code proposed in eq. (6) with $2M$-BPPM.

IV. Simulations and results

Simulations are performed over the IEEE 802.15.3a channel model recommendation CM2 that corresponds to non line of sight situations [7]. Fig. 1 compares the proposed code with the best previously known totally-real code [4]. The latter can be obtained from eq. (6) by setting $\Omega = 2$. This choice maximizes the coding gain over all constellations. The receiver is equipped with 2 antennas and a 1 finger Rake. The superiority of the proposed scheme is obvious in all situations. In comparison with the single-antenna systems, the 2×2 scheme doubles the data rate and enhances the performance.

In Fig. 2, the proposed code is compared with the Golden code in the case where the UWB receivers are equipped with IQ front ends. Results show that our totally real construction shows exactly the same performance as [1]. Results with 4-PPM-4-PAM are shown since the coding gain associated with this constellation is equal to $2/\sqrt{5}$.

V. Conclusion

A 2×2 totally-real ST code suitable for low-cost UWB terminals using multi-dimensional constellations is proposed. The information lossless scheme achieves full rate and full diversity with all M-PPM-M'-PAM constellations for $M > 2$. The proposed code achieves the optimal coding gain with $2M$-BPPM for $M \geq 4$ and with M-PPM for $M \geq 3$.

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