Towards a Better Comprehension of Decode-and-Forward Buffer-Aided Relaying: Case Study of a Single Relay

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***Abstract*—This work considers the problem of decode-and-forward (DF) buffer-aided (BA) relaying with a single relay. For this scenario, a Markov chain analysis is tractable resulting in closed-form performance measures that can shed more light on the capabilities of the more general BA networks. In particular, a novel generic relaying scheme is proposed and this scheme encompasses many of the existing single relay schemes as special cases. The proposed scheme is controlled by a single parameter and the paper highlights on the impact of this parameter on the triad of diversity gain, coding gain and queuing delay that can be contemplated.**

***Index Terms*—Relaying, buffer, dual-hop, outage probability, diversity order, coding gain, queuing delay, asymptotic analysis.**

I. Introduction

Cooperative relaying has been extensively studied where information sources (S) communicate with their destinations (D) through relays (R). In this context, equipping the relays with buffers constitutes an additional degree of freedom capable of leveraging the reliability and throughput of wireless networks [1]. The literature on buffer-aided (BA) relaying is extensive and revolves mainly around the relay selection strategies in half-duplex (HD) networks [2] with amplify-and-forward (AF) relaying [3] and decode-and-forward (DF) relaying [4]–[10].

The max-link DF BA scheme was proposed in [4] where the strongest link among all available S-R and R-D links is selected. For infinitely large buffer sizes, the max-link scheme achieves a diversity order of $2K$ with K relays. This enhanced diversity gain comes at the expense of an average packet delay (APD) of $KL+1$ where L is the buffer size. These findings hold for symmetrical networks where all links are independent and identically distributed (iid). A priority-based max-link scheme was proposed in [5] where three classes of priority were considered; namely relays with full, empty and neither full nor empty buffers. The diversity order was also proven to be equal to $2K$ for large values of L in the case of symmetrical networks. [5] results in reduced outage probabilities (OP) compared to the max-link scheme especially in the case of asymmetrical networks. In an attempt to reduce the APD of the max-link protocol, priority was given to the R-D links in [6]. This resulted in reducing the APD to 2 for quasi-symmetrical networks where the S-R links are iid and the R-D links are iid. This enhancement in the APD came at the expense of a deteriorated OP performance in the cases of symmetrical networks and quasi-symmetrical networks where the R-D links are stronger than the S-R links. A BA relay selection scheme taking into account both the channel quality and buffer state was proposed in [7]. For symmetrical networks, this scheme achieves a diversity order of $2K$ with finite buffer sizes along with an improved APD of $2K+2$. A two-stage relay selection strategy was advised in [8]. For symmetrical networks, [8] demonstrated the capability of achieving the diversity order of $2K$ for $L \geq 3$. Finally, a balancing BA scheme was analyzed in [9] targeting to keep the number of packets at each buffer the closest possible to $L/2$ in symmetrical networks. Three-node DF BA relaying was considered in [11], [12] with the objective of maximizing the throughput over a communication session that extends over an infinite number of time slots. The analysis in [11], [12] targeted mainly infinite-size buffers and, for finite-size buffers, extremely large buffer sizes were considered in order to avoid buffer overflow.

Surveying the extensive literature on DF BA relaying [4]–[9], the following observations can be made. (i): For networks with an arbitrary number of relays, no relaying scheme is unconditionally better than the others. (ii): For single-relay networks, many of the existing schemes will become equivalent. (iii): Except for [6], the diversity order analysis is often carried out for the symmetrical networks which, in some cases, might be misleading since the performance of the BA schemes is highly dependent on the network topology [5]–[7], [9].

Motivated by the above observations, this work targets a comprehensive analysis of DF BA systems. In order to reach insightful results, this paper considers the special case of one relay where the mathematical analysis is more tractable. In fact, the complexity of the analysis with an arbitrary number of relays inflicted some assumptions on the system model and restrained the number of performance measures that were derived in closed-form. For example, the diversity order analysis was limited to symmetrical networks in many references [4], [5], [7]–[9] while the APD expressions were not provided for many relaying protocols [5], [8], [9]. Finally, unlike [11], [12], the presented relaying scheme and performance analysis hold for any value of the buffer size and, in particular, for practically small sizes. Moreover, the theoretical analysis in [11], [12] is valid for infinitely long transmission sessions where the transient effects resulting from filling the buffer at the beginning of transmission and emptying it at the end of transmission are negligible. In this context, the proposed slot-by-slot relaying scheme takes the aforementioned tran-

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TABLE I

SELECTION STRATEGIES WITH ONE RELAY FOR $0 < l < L$ WHEN THE S-R AND R-D LINKS ARE NOT IN OUTAGE.

Scheme	[4], [5]	[6], ThBA(0)	[7], ThBA(1)	[8], [9], ThBA($L/2$)
Selection Strategy	$\begin{cases} \lvert h_{\rm SR}\rvert^2 > \lvert h_{\rm RD}\rvert^2, & \text{Rx;} \\ \lvert h_{\rm SR}\rvert^2 < \lvert h_{\rm RD}\rvert^2, & \text{Tx.} \end{cases}$	Tx	$\begin{cases} l \leq 1, & \text{Rx;} \\ l > 1, & \text{Tx.} \end{cases}$	$\begin{cases} l \leq L/2, & \text{Rx;} \\ l > L/2, & \text{Tx.} \end{cases}$

sient effects into consideration since these effects cannot be neglected when the buffer size is not very large.

The contributions of this work are twofold. First, a novel DF BA scheme for single-relay networks is proposed. Second, the achievable OP and APD are derived in closed-form followed by an asymptotic analysis that is culminated by evaluating the diversity gain, coding gain and asymptotic delay. A special emphasis in the presented performance analysis is dedicated to the effect of the network topology on the previously delineated performance metrics. Unlike [4]–[9] where the relaying rule is invariable, the proposed scheme is parameterized by a single variable and it encompasses these schemes as special cases. This paper highlights how this parameter can be selected to minimize the OP, minimize the APD or achieve adequate tradeoffs between OP and APD.

II. SYSTEM MODEL AND RELAYING STRATEGY

Consider a three node network composed of a source (S), destination (D) and relay (R) equipped with a single antenna each. We assume that no direct link is available between S and D and, hence, S communicates with D through R that is equipped with a buffer of size L. The relay operates in HD mode and, hence, can not transmit and receive simultaneously.

Consider a Rayleigh block fading channel model where $h_{\rm SR}$ and $h_{\rm RD}$ denote the channel coefficients of the S-R and R-D links, respectively. These channel coefficients are assumed to be circularly symmetric complex Gaussian distributed random variables with zero mean and variances $\Omega_{\rm SR}$ and $\Omega_{\rm RD}$, respectively [7]. Finally, it is assumed that the signals received at R and D are corrupted by an additive white Gaussian noise (AWGN) with zero mean and unit variance.

For a fixed target rate r_0 (in bits per channel use (BPCU)), the S-R link is in outage with the following probability [7]:

$$p_{\rm SR} = \Pr\left\{\frac{1}{2}\log_2\left(1+\bar{\gamma}|h_{\rm SR}|^2\right) \leq r_0\right\} = 1 - e^{-\frac{2^{2r_0}-1}{\Omega_{\rm SR}\bar{\gamma}}}, \quad (1)$$

where $\bar{\gamma}$ is the average transmit signal-to-noise ratio (SNR). Similarly, the outage probability along the R-D link is given by $p_{\rm RD} = 1 - e^{-\frac{2^{2r_0}-1}{\Omega_{\rm RD}\bar{\gamma}}}$.

We denote by l the number of packets stored in the buffer. Irrespective of the implemented relaying strategy, when $l = 0$ (resp. $l = L$), the buffer is empty (resp. full) and no packets can be transmitted along the R-D (resp. S-R) link. When $0 < l < L$, both the S-R and R-D links are available and the protocol needs to select the link to be activated resulting in the following cases. (i): If both the S-R and R-D links are in outage, then no packets can be transmitted or received. (ii): If the S-R link is in outage while the R-D link is not in outage, R switches to the transmission mode. (iii): If the S-R link is not in outage while the R-D link is in outage, R switches to the reception mode. (iv): When both the S-R and R-D links are not in outage, a selection strategy among these links must be implemented. The first three cases remove any uncertainty pertaining to the link selection. Consequently, the differentiation among the relaying protocols arises in the way they handle the fourth case. In this paper, we propose the following threshold-based BA strategy that is a function of a buffer occupancy threshold level l_{th} as follows:

$$\text{ThBA}(l_{th}): \quad \begin{cases} l \leq l_{th}, & \text{Rx;} \\ l > l_{th}, & \text{Tx.} \end{cases} \quad ; \quad l_{th} \in \{0, \ldots, L\}, \quad (2)$$

where "Rx" means that R chooses to receive and "Tx" means that R chooses to transmit. The rationale behind (2) is to incentivize R to receive when the number of stored packets is small and to transmit otherwise.

In the special case of one relay with $0 < l < L$ when both the S-R and R-D links are not in outage, many of the existing selection schemes will become equivalent to each other while other schemes will follow as special cases of the proposed scheme as summarized in Table I. In particular, [6] is equivalent to ThBA(0), [7] is equivalent to ThBA(1) while [8], [9] are equivalent to ThBA$(L/2)$ (when L is even).

III. PERFORMANCE ANALYSIS

In this section, a Markov chain analysis is carried out for evaluating the steady-state distribution, OP and APD. A state of the Markov chain is defined as the number of stored packets resulting in $L+1$ possible states. Denote by $t_{l,l'}$ the transition probability of going from state l to state l'. For all considered schemes [4]–[9] (with one relay) as well as the proposed scheme, the following relations hold:

$$\begin{cases} t_{0,0} = p_{\rm SR}, \\ t_{0,1} = 1 - p_{\rm SR}. \end{cases} ; \quad \begin{cases} t_{L,L} = p_{\rm RD}, \\ t_{L,L-1} = 1 - p_{\rm RD}. \end{cases} , \quad (3)$$

where an empty (resp. full) buffer remains empty (resp. full) if the S-R (resp. R-D) link is in outage since no transmission can take place along the R-D (resp. S-R) link; otherwise, the number of packets will increase (resp. decrease) by one.

The steady-state probability distribution is determined by $\{\pi_l\}_{l=0}^{L}$ where π_l stands for the probability of having l packets in the buffer at steady-state. The system is in outage when no packets can be communicated along its links [4]–[9]:

$$P_{out} = \pi_0 p_{\rm SR} + \sum_{l=1}^{L-1} \pi_l p_{\rm SR} p_{\rm RD} + \pi_L p_{\rm RD}, \quad (4)$$

since when the buffer is empty (resp. full), no packets can be transmitted if the S-R (resp. R-D) link is in outage. Otherwise, the outage of the S-R and R-D links will incur a system outage.

Following from [6], the APD can be calculated as follows:

$$APD = 1 + \frac{2\bar{L}}{1 - P_{out}}, \quad (5)$$

where $\bar{L}=\sum_{l=0}^{L} l\pi_l$ stands for the average queue length.

The cases of an empty or full buffer were considered in (3) since R can only receive or transmit, respectively. The subsequent analysis considers the case where the buffer is neither empty nor full. In this case, R has the option to receive or transmit according to the implemented relaying strategy.

A. [4]–[6], ThBA(0) and ThBA(L):

This paper defines a birth-death Markov process with birth probability λ and death probability μ, denoted by BD(λ,μ), as a discrete Markov chain where (3) is satisfied for $l=0$ and $l=L$ while $(t_{l,l+1},t_{l,l-1})=(\lambda,\mu)$ for $l=1,\dots,L-1$. The schemes in [4]–[6] and the proposed scheme for $l_{th}=0$ (preferred Tx) and $l_{th}=L$ (preferred Rx) all correspond to birth-death processes where only the parameters λ and μ vary from one scheme to another.

Proposition 1: The steady-state distribution of the process BD(λ,μ) is given by:

$$\begin{cases} \pi_0=\frac{\lambda(r-1)(1-p_{\text{SR}})^{-1}}{(r^L-1)+(r-1)\left[\frac{\lambda}{1-p_{\text{SR}}}-1+\frac{\mu}{1-p_{\text{RD}}}r^L\right]}, \\ \pi_l=\frac{1-p_{\text{SR}}}{\lambda}r^l\pi_0 \ ; \ l=1,\dots,L-1, \\ \pi_L=\frac{1-p_{\text{SR}}}{1-p_{\text{RD}}}r^{L-1}\pi_0, \end{cases} \quad ; \quad r\triangleq\frac{\lambda}{\mu}. \tag{6}$$

Proof: The proof follows from solving the following balance equations while taking into consideration that $\sum_{l=0}^{L}\pi_l=1$. (i): $(1-p_{\text{SR}})\pi_0=\mu\pi_1$. (ii): $(\lambda+\mu)\pi_1=(1-p_{\text{SR}})\pi_0+\mu\pi_2$. (iii): $(\lambda+\mu)\pi_l=\lambda\pi_{l-1}+\mu\pi_{l+1}$ for $l=2,\dots,L-2$. (iv): $(\lambda+\mu)\pi_{L-1}=\lambda\pi_{L-2}+(1-p_{\text{RD}})\pi_L$. ∎

1) [4], [5]: From Table I, for the max-link scheme [4], [5], $\lambda=(1-p_{\text{SR}}p_{\text{RD}})\alpha$ and $\mu=(1-p_{\text{SR}}p_{\text{RD}})(1-\alpha)$ where $\alpha\triangleq\Pr\left(|h_{\text{SR}}|^2>|h_{\text{RD}}|^2\right)=\frac{\Omega_{\text{SR}}}{\Omega_{\text{SR}}+\Omega_{\text{RD}}}$ through direct calculations. Replacing the values of λ and μ in (6) results in the following asymptotic value of π_0 as the SNR tends to infinity:

$$\pi_0=\frac{\alpha(r-1)}{(r^L-1)\left[1+(1-\alpha)(r-1)\right]} \quad ; \quad r=\frac{\alpha}{1-\alpha}, \tag{7}$$

where the probabilities $1-p_{\text{SR}}$, $1-p_{\text{RD}}$ and $1-p_{\text{SR}}p_{\text{RD}}$ all tend to 1 asymptotically.

For the symmetrical case ($\Omega_{\text{SR}}=\Omega_{\text{RD}}$), $\alpha=\frac{1}{2}$ implying that $r=1$. Taking the limit of (7) as r tends to 1 and replacing in (6) results in:

$$\pi_0=\pi_L=\frac{1}{2L} \quad ; \quad \pi_1=\dots=\pi_{L-1}=\frac{1}{L}, \tag{8}$$

in coherence with [4]. Replacing (8) in (4)-(5) results in:

$$P_{out}=\frac{1}{2L}p_{\text{SR}}+\frac{L-1}{L}p_{\text{SR}}p_{\text{RD}}+\frac{1}{2L}p_{\text{RD}} \quad ; \quad APD=L+1. \tag{9}$$

Equation (9) shows that the APD of the max-link scheme increases linearly with L in coherence with the existing literature. Moreover, for $L\to+\infty$, $P_{out}\to p_{\text{SR}}p_{\text{RD}}$ that behaves asymptotically as $\bar{\gamma}^{-2}$ implying that the diversity order is doubled in this case unlike the case where L is finite.

The new findings correspond to the asymmetrical case where the replacement of (7) in (6) results in $\pi_0=\frac{2\alpha-1}{2(1-\alpha)(r^L-1)}$, $\pi_L=r^{L-1}\pi_0$ and $\pi_l=\frac{r^l}{\alpha}\pi_0$ for $l=1,\dots,L-1$. Since α and r do not depend on the SNR asymptotically, this result shows that the Markov chain can be in any of the $L+1$ states with a probability not tending to zero and, hence, no dominant states can be observed in this case. Replacing the above probabilities in (4) results in:

$$P_{out}=\frac{2\alpha-1}{2(1-\alpha)(r^L-1)}\left[p_{\text{SR}}+\frac{r^{L-1}-1}{2\alpha-1}p_{\text{SR}}p_{\text{RD}}+r^{L-1}p_{\text{RD}}\right]. \tag{10}$$

For finite values of L, (10) behaves asymptotically as $\bar{\gamma}^{-1}$ showing that there is no improvement in the diversity order. Consider now the case $L\to+\infty$. If R is closer to S, then $\Omega_{\text{SR}}>\Omega_{\text{RD}}$ implying that $\alpha>\frac{1}{2}$ and $r>1$. Therefore, $P_{out}\to\frac{2\alpha-1}{2\alpha}p_{\text{RD}}$ (since $r^L\to\infty$ and $r^{L-1}\to\infty$) implying that the diversity order is equal to 1. If R is closer to D, then $\alpha<\frac{1}{2}$ and $r<1$ implying that r^L and r^{L-1} will tend to zero. Consequently, $P_{out}\to\frac{2\alpha-1}{2(\alpha-1)}p_{\text{SR}}$ implying a diversity order of 1. As a conclusion, it has been proven that [4], [5] can not improve the diversity order in asymmetrical networks even with infinite buffer sizes. In this context, the max-link scheme is capable of doubling the diversity order only for symmetrical networks with infinite buffer sizes. Evidently, the above findings hold for the special case of one relay, but it is excepted that they will hold in the general case as well.

Carrying out a similar analysis to evaluate the asymptotic APD results in:

$$APD=\begin{cases} 2\left[L-\frac{1-\alpha}{2\alpha-1}\right], & \text{R is closer to S;} \\ 2\frac{1-\alpha}{1-2\alpha}, & \text{R is closer to D.} \end{cases} \tag{11}$$

Equation (11) shows that the APD increases linearly with L only if R is closer to S since the arrival rate will exceed the departure rate resulting in a more congested buffer in this case. On the other hand, when R is closer to D, the packets arriving at R's buffer will have a higher chance of exiting this buffer resulting in an APD that is independent of L. It can be easily proven that the APD is always smaller in the second case where the buffer is not congested. These conclusions are novel and were not reported before.

2) [6], ThBA(0): For these schemes $\lambda=(1-p_{\text{SR}})p_{\text{RD}}$ and $\mu=1-p_{\text{RD}}$ where R chooses to transmit whenever the R-D link is not in outage. In this case, $r=\frac{\lambda}{\mu}\to p_{\text{RD}}$ asymptotically. Since r^L will tend to zero for $L\geq 2$, then the probability π_0 in (6) will tend asymptotically to:

$$\pi_0=\frac{p_{\text{RD}}}{\frac{r^L-1}{r-1}+(p_{\text{RD}}-1)}\to\frac{1}{2}, \tag{12}$$

implying that $\pi_l\to\frac{1}{2}p_{\text{RD}}^{l-1}$ for $l=1,\dots,L-1$ and $\pi_L\to\frac{1}{2}p_{\text{RD}}^{L-1}$ from (6). Consequently, $\pi_0\to\frac{1}{2}$ and $\pi_1\to\frac{1}{2}$ while all other probabilities will tend to zero asymptotically. This scheme that privileges transmission from R will relax the buffer occupancy resulting in a buffer that is either empty or has one packet all of the time where these two states dominate at equilibrium. Replacing in (4)-(5) results in:

$$P_{out}=\frac{1}{2}p_{\text{SR}}(1+p_{\text{RD}})\to\frac{1}{2}p_{\text{SR}} \quad ; \quad APD=2. \tag{13}$$

Equation (13) shows that [6] and ThBA(0) are not capable of enhancing the diversity order neither in symmetrical nor in asymmetrical networks even with infinite buffer sizes. The advantage of these schemes resides in the appealing APD value of 2 independently from the network topology and the buffer size in coherence with [6].

3) ThBA(L): For this preferred-reception scheme, $\lambda = 1 - p_{\text{SR}}$ and $\mu = p_{\text{SR}}(1 - p_{\text{RD}})$ implying that $r \to \frac{1}{p_{\text{SR}}}$. Carrying out an asymptotic analysis with large values of r shows that $\pi_{L-1} \to \frac{1}{2}$ and $\pi_L \to \frac{1}{2}$ while all other probabilities will tend to zero. Hence, the states $L-1$ and L will dominate asymptotically and this scheme results in a congested buffer. Replacing in (4)-(5) results in:

$$P_{out} = \frac{1}{2} p_{\text{RD}}(1 + p_{\text{SR}}) \to \frac{1}{2} p_{\text{RD}} \quad ; \quad APD = 2L, \tag{14}$$

rendering the choice $l_{th} = L$ not appealing neither from a diversity order nor from a delay point of view. From (13) and (14), the only advantage of ThBA(L) resides in a smaller OP where R is closer to D.

B. [7]–[9] and ThBA(l_{th}) for $0 < l_{th} < L$:

When $l_{th} \neq 0$ and $l_{th} \neq L$, the Markov chain is not equivalent to a birth-death process. Following from (2), for $l \leq l_{th}$, $t_{l,l+1} = 1 - p_{\text{SR}} \triangleq \lambda_1$ and $t_{l,l-1} = p_{\text{SR}}(1 - p_{\text{RD}}) \triangleq \mu_1$ following from an analysis similar to that presented in Section III-A3. For $l > l_{th}$, $t_{l,l+1} = (1 - p_{\text{SR}})p_{\text{RD}} \triangleq \lambda_2$ and $t_{l,l-1} = 1 - p_{\text{RD}} \triangleq \mu_2$ following from an analysis similar to that presented in Section III-A2.

Proposition *2:* The steady-state probability distribution of the Markov chain is given by:

$$\pi_l = r^l \pi_0 \times \begin{cases} \frac{1}{p_{\text{SR}}^l}, & l = 1, \dots, l_{th}; \\ \frac{1}{p_{\text{SR}}^{l_{th}}}, & l = l_{th} + 1; \\ \frac{p_{\text{RD}}^{l-(l_{th}+1)}}{p_{\text{SR}}^{l_{th}}}, & l = l_{th} + 2, \dots, L. \end{cases} \quad ; \; r \triangleq \frac{1 - p_{\text{SR}}}{1 - p_{\text{RD}}}, \tag{15}$$

where:

$$\pi_0 = \left[1 + \frac{\frac{r}{p_{\text{SR}}} - \left(\frac{r}{p_{\text{SR}}}\right)^{l_{th}+1}}{1 - \frac{r}{p_{\text{SR}}}} + \frac{r^{l_{th}+1}}{p_{\text{SR}}^{l_{th}}} + \frac{1}{p_{\text{RD}}(p_{\text{SR}}p_{\text{RD}})^{l_{th}}} \frac{(rp_{\text{RD}})^{l_{th}+2} - (rp_{\text{RD}})^{L+1}}{1 - rp_{\text{RD}}} \right]^{-1}. \tag{16}$$

Proof: The solution in (15)-(16) follows from solving the following balance equations while taking into consideration that $\sum_{l=0}^{L} \pi_l = 1$. (i): $(1 - p_{\text{SR}})\pi_0 = \mu_1 \pi_1$. (ii): $(\lambda_1 + \mu_1)\pi_l = \lambda_1 \pi_{l-1} + \mu_1 \pi_{l+1}$ for $l = 1, \dots, l_{th} - 1$. (iii): $(\lambda_1 + \mu_1)\pi_{l_{th}} = \lambda_1 \pi_{l_{th}-1} + \mu_2 \pi_{l_{th}+1}$. (iv): $(\lambda_2 + \mu_2)\pi_{l_{th}+1} = \lambda_1 \pi_{l_{th}} + \mu_2 \pi_{l_{th}+2}$. (v): $(\lambda_2 + \mu_2)\pi_l = \lambda_2 \pi_{l-1} + \mu_2 \pi_{l+1}$ for $l = l_{th} + 2, \dots, L - 1$. ■

Since for large SNRs $\frac{r}{p_{\text{SR}}} \gg 1$ and $rp_{\text{RD}} \ll 1$, (16) tends to the following asymptotic value:

$$\pi_0 = \frac{p_{\text{SR}}^{l_{th}}}{p_{\text{SR}}^{l_{th}} + r^{l_{th}}\left[1 + r + r^2 p_{\text{RD}}\right]}. \tag{17}$$

From (15), $\pi_l = \left(\frac{r}{p_{\text{SR}}}\right)^l \pi_0$ for $l = 1, \dots, l_{th}$ implying, from (17), that $\pi_{l_{th}-1}$ is proportional to p_{SR}, $\pi_{l_{th}}$ tends to a constant while the other probabilities (for $l = 1, \dots, l_{th} - 2$) will be proportional to higher powers of p_{SR} and, hence, can be neglected. Similarly, $\pi_{l_{th}+1}$ tends to a constant, $\pi_{l_{th}+2}$ is proportional to p_{RD} while $\{\pi_l\}_{l=l_{th}+3}^{L}$ are proportional to higher powers of p_{RD}.

For APD calculations, it's sufficient to consider only $\pi_{l_{th}}$ and $\pi_{l_{th}+1}$ that will tend to $1/2$ resulting, from (5), in:

$$APD = 2(l_{th} + 1) \quad ; \quad l_{th} \in \{1, \dots, L - 1\}. \tag{18}$$

Regarding the OP, replacing the highest four probabilities $\{\pi_l\}_{l=l_{th}-1}^{l_{th}+2}$ in (4) results in:

$$P_{out} = \begin{cases} p_{\text{SR}}p_{\text{RD}} + \frac{1}{2}p_{\text{SR}}^2, & l_{th} = 1; \\ p_{\text{SR}}p_{\text{RD}}, & l_{th} = 2, \dots, L - 3; \\ p_{\text{SR}}p_{\text{RD}} + \frac{1}{2}p_{\text{RD}}^2, & l_{th} = L - 2; \\ \frac{1}{2}p_{\text{RD}}, & l_{th} = L - 1. \end{cases} \tag{19}$$

Equations (13), (14) and (19) show that ThBA(l_{th}) is capable of doubling the diversity order for all values of l_{th} in $\{1, \dots, L-2\}$ and, in particular, [7]–[9] are all efficient in maximizing the diversity gain. Among these values, the choice $l_{th} = 1$ minimizes the APD following from (18). The choice $l_{th} = 2$ is also viable since, compared with $l_{th} = 1$, it reduces the OP at the expense of increasing the APD. Any other choice of l_{th} in $\{3, \dots, L - 2\}$ will suffer from an increased APD compared to both ThBA(1) and ThBA(2) while achieving an OP that is, at best, the same as ThBA(2). As a conclusion, the best diversity-enhancing relaying solutions are ThBA(1) and ThBA(2). While the first choice minimizes the APD to 4, the second alternative minimizes the OP with an APD smaller than that of ThBA(l_{th}) for all values of l_{th} exceeding 2.

Direct calculations following from (1) and (19) show that the coding gain of ThBA(2) over ThBA(1) is:

$$G = 5 \log_{10} \left(1 + \frac{\Omega_{\text{RD}}}{2\Omega_{\text{SR}}}\right) \quad \text{[dB]}, \tag{20}$$

and, hence, this gain is higher when R is closer to D.

IV. Numerical Results

In what follows, r_0 and L are fixed to $r_0 = 1$ BPCU and $L = 8$. Figures 1 and 2 show the OP and APD, respectively, for $(\Omega_{\text{SR}}, \Omega_{\text{RD}}) = (4, 1)$ (R is closer to S). Fig. 3 and Fig. 4 target the case where R is closer to D with $(\Omega_{\text{SR}}, \Omega_{\text{RD}}) = (1, 4)$. All presented results highlight on the accuracy of the provided theoretical analysis and on the validity of the derived OP and APD asymptotic expressions.

Fig. 1 shows that, among the considered schemes, only ThBA(1) and ThBA(2) achieve a diversity order of 2 with a slight advantage for ThBA(2) that outperforms ThBA(1) by 0.25 dB in coherence with (20). Comparing ThBA(0) and ThBA(L) shows that the former scheme achieves a smaller OP, in coherence with (13)-(14), since $p_{\text{SR}} < p_{\text{RD}}$ in this case. Results in Fig. 2 validate (11), (13), (14) and (18). The APD advantage of ThBA(l_{th}) for small values of l_{th} over the max-link scheme is evident in this scenario where the max-link scheme selects the S-R link more often thus contributing to increasing the number of stored packets and, hence, the APD.

Similar OP trends can be observed in Fig. 3 but now ThBA(L) outperforms ThBA(0) (since $p_{\text{RD}} < p_{\text{SR}}$) while ThBA$(2)$ outperforms ThBA(1) by a significant coding gain of

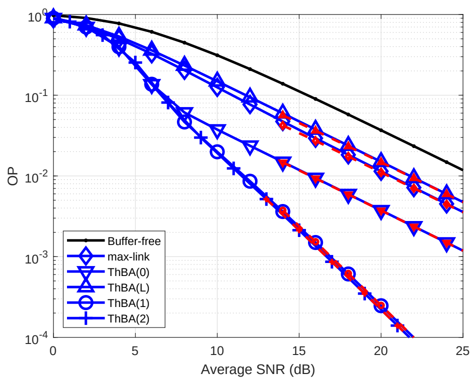

Fig. 1. OP for $\Omega_{\mathrm{SR}} = 4$ and $\Omega_{\mathrm{RD}} = 1$. Solid and dashed lines correspond to the exact and asymptotic values, respectively.

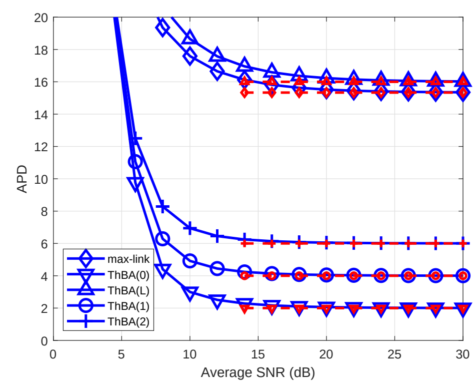

Fig. 2. APD for $\Omega_{\mathrm{SR}} = 4$ and $\Omega_{\mathrm{RD}} = 1$. Solid and dashed lines correspond to the exact and asymptotic values, respectively.

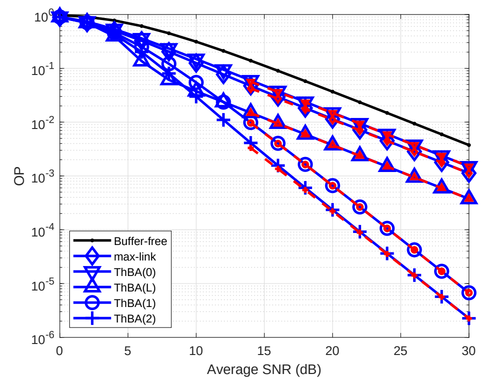

Fig. 3. OP for $\Omega_{\mathrm{SR}} = 1$ and $\Omega_{\mathrm{RD}} = 4$. Solid and dashed lines correspond to the exact and asymptotic values, respectively.

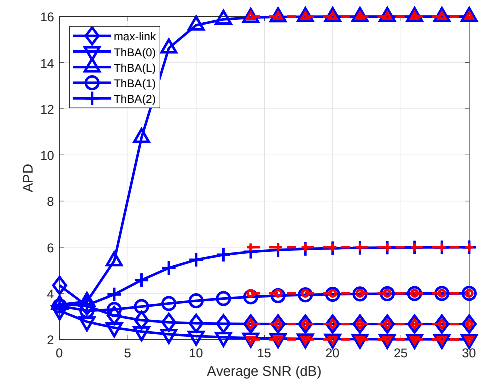

Fig. 4. APD for $\Omega_{\mathrm{SR}} = 1$ and $\Omega_{\mathrm{RD}} = 4$. Solid and dashed lines correspond to the exact and asymptotic values, respectively.

2.38 dB in coherence with (20) where G increases with $\frac{\Omega_{\mathrm{RD}}}{\Omega_{\mathrm{SR}}}$. From Fig. 4, ThBA$(0)$ and ThBA(L) achieve the best and worst APD performance, respectively, while the asymptotic APD of the max-link scheme assumes an acceptable value of 2.67 in coherence with (11) where $\alpha = \frac{1}{5}$. The coding gain of ThBA(2) over ThBA(1) is associated with an increase in the asymptotic APD from 4 to 6.

V. Conclusion

This paper proposed and analyzed a novel threshold-based relaying scheme for HD BA systems with a single relay. This work studied the impact of the threshold parameter on the diversity order, outage probability and queuing delay and suggested convenient choices of this parameter. While this work sheds more light on the performance of DF BA networks, future work must consider the extension of the proposed scheme to the general case of an arbitrary number of relays.

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