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Exploiting the relays' backup RF antennas for enhanced FSO cooperative communications

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Abstract: This paper investigates a novel method for boosting the performance of parallel-relaying decode-and-forward (DF) cooperative free space optical (FSO) networks. The proposed solution takes advantage from the presence of the radio frequency (RF) links that are often established to backup the FSO links in practical systems. In this context, the low speed RF links are used for carrying the information packets that have not been delivered along the direct FSO link for the sake of sharing these packets among the relays in a simple and efficient manner. This packet sharing enhances the chances for correct detection at the relays thus increasing the number of relays that will participate in the relay-destination transmission phase. An exact outage probability analysis is carried out in the case of gamma-gamma FSO fading channels with pointing errors and of Rician/Rayleigh RF fading channels. At a second time, the cut-set method is applied for deriving a simple and tractable upper-bound that is useful for evaluating the asymptotic performance and diversity gain.

OCIS codes: (060.4510) Optical communications; (060.2605) Free-space optical communication; (010.1330) Atmospheric turbulence; (290.5930) Scintillation.

References and links

1. S. Bloom and W. S. Hartley, "The last-mile solution: hybrid FSO radio," Whitepaper, AirFiber Inc, (2002).
2. A. Akbulut, H. G. Ilk, and F. Ari, "Design, availability and reliability analysis on an experimental outdoor FSO/RF communication system," in Proc. 7th IEEE Int. Conf. on Transparent Optical Networks (ICTON), 403–406 (2005).
3. M. Usman, H.-C. Yang, and M.-S. Alouini, "Practical switching-based hybrid FSO/RF transmission and its performance analysis," IEEE Photonics J. **6**(5), 1–13 (2014).
4. W. Zhang, S. Hranilovic, and C. Shi, "Soft-switching hybrid FSO/RF links using short-length raptor codes: design and implementation," IEEE J. Sel. Areas Commun. **27**(9), 1698–1708 (2009).
5. A. AbdulHussein, A. Oka, T. T. Nguyen, and L. Lampe, "Rateless coding for hybrid free-space optical and radio-frequency communication," IEEE Trans. on Wireless Commun. **9**(3), 907–913 (2010).
6. E. Zedini, H. Soury, and M.-S. Alouini, "On the performance analysis of dual-hop mixed FSO/RF systems," IEEE Trans. on Wireless Commun. **15**(5), 3679–3689 (2016).
7. M. I. Petkovic, A. M. Cvetkovic, G. T. Djordjevic, and G. K. Karagiannidis, "Partial relay selection with outdated channel state estimation in mixed RF/FSO systems," J. Lightwave Technol. **33**(13), 2860–2867 (2015).
8. V. Jamali, D. S. Michalopoulos, M. Uysal, and R. Schober, "Link allocation for multiuser systems with hybrid RF/FSO backhaul: Delay-limited and delay-tolerant designs," IEEE Trans. on Wireless Commun. **15**(5), 3281–3295 (2016).
9. M. Safari and M. Uysal, "Relay-assisted free-space optical communication," IEEE Trans. on Wireless Commun. **7**(12), 5441 – 5449 (2008).
10. C. Abou-Rjeily, "Performance analysis of FSO communications with diversity methods: Add more relays or more apertures?" IEEE J. Sel. Areas Commun. **33**(9), 1890 – 1902 (2015).
11. C. Abou-Rjeily and Z. Noun, "Impact of inter-relay cooperation on the performance of FSO systems with any number of relays," IEEE Trans. on Wireless Commun. **15**(6), 3796–3809 (2016).
12. H. Sandalidis, T. Tsiftsis, and G. Karagiannidis, "Optical wireless communications with heterodyne detection over turbulence channels with pointing errors," J. Lightwave Technol. **27**(20), 4440–4445 (2009).
13. W. Limpakom, Y.-D. Yao, and H. Man, "Outage probability analysis of wireless relay and cooperative networks in Rician fading channels with different K-factors," in Proc. 69th IEEE Vehicular Tech. Conf. (VTC-Spring), 1–5 (2009).
14. R. Billinton and R. N. Allan, *Reliability evaluation of power systems*. Springer Science & Business Media, 2nd ed. (Springer, 2013).

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15. J. Fussell and W. Vesely, "New methodology for obtaining cut sets for fault trees," Trans. Amer. Nucl. Soc. **15**, 262–263 (1972).
16. A. Rauzy, "Toward an efficient implementation of the MOCUS algorithm," IEEE Trans. on Reliability **52**(2), 175–180 (2003).

1. Introduction

Commercially available Free Space Optics (FSO) transceivers are often equipped with a Radio Frequency (RF) backup system to be used when the FSO link is inoperative [1]. In fact, while FSO links advantageously provide high data rates, they become practically unavailable under severe weather conditions like fog or snow where the attenuation reaches the order of hundreds dB/km [2]. This renders hybrid RF/FSO solutions particularly appealing since they take advantage from the high data rate of the FSO links and the high reliability of the RF links [2–5]. A hard-switching hybrid RF/FSO system was analyzed in [3] where preference is given to the high speed FSO link as long as the link quality is above a certain threshold; if the quality of the FSO link falls below acceptable levels, the system resorts to the lower speed RF link. While hard-switching involves the transmission along a single channel at a time, a soft-switching technique was proposed in [4] where packets are transmitted simultaneously over both links. In this case, a channel coding scheme that adapts to the conditions along the FSO and RF links is used to coordinate data transmission. A similar solution was considered in [5] where a single rate-less code is applied at the transmitter side followed by de-multiplexing the encoded bits into two streams to be communicated simultaneously along the FSO and RF links.

In addition to the hybrid RF/FSO point-to-point communication systems [3–5], dual-hop mixed RF/FSO relaying systems attracted an increased attention recently [6–8]. For such systems, an intermediate relay ensures the wireless connectivity between communicating nodes with no direct Line-of-Sight (LOS). In this case, the relay communicates with a number of users along RF links and forwards the information to the destination along an FSO link. Dual-hop mixed RF/FSO systems constitute very powerful solutions for bridging the connectivity gap between a backbone network and a last-mile network where the information from a number of users can be multiplexed along a high speed FSO link. Single-relay and multi-relay Amplify-and-Forward (AF) mixed RF/FSO systems were analyzed in [6] and [7], respectively. Multiuser single-relay Decode-and-Forward (DF) systems were analyzed in [8] where the last-mile links are RF while the backbone link is a hybrid RF/FSO link.

On the other hand, FSO cooperative diversity was investigated extensively where a group of relays (R's) assist a source (S) in its communication with a destination (D). The cooperative solutions include the two-phase (S-R and R-D) parallel-relaying AF and DF schemes [9, 10] and the three-phase (S-R, R-R and R-D) DF strategies [11].

In this paper, we propose and analyze a novel three-phase mixed RF/FSO scheme where communications take place along the S-D, S-R and R-D FSO links and along the R-R RF links. The proposed approach is motivated by the observation that the RF links operating at lower data rates can be used to communicate the packets that are not delivered along the S-D link. In other words, the RF links are used as a backup system for the sake of sharing the packets between the relays and establishing the inter-relay cooperation phase. The main differences between the proposed scheme and the inter-relay cooperation strategies in [11] are as follows. (i): In [11], the R-R links are FSO while in this work they are RF links. (ii): The implication of establishing FSO R-R links in [11] is that only subsequent relays can be connected; therefore, the R-R communications take place along a total of $N - 1$ links where N is the number of relays. Since RF transmissions have a broadcast nature, the packet transmitted by one relay can be overheard by all other relays. Consequently, any two relays are interconnected resulting in a total of $\binom{N}{2}$ RF R-R links that can be exploited. This not only alters the cooperation protocol, but also complicates the outage analysis of the network. (iii): While in [11] all packets are transmitted

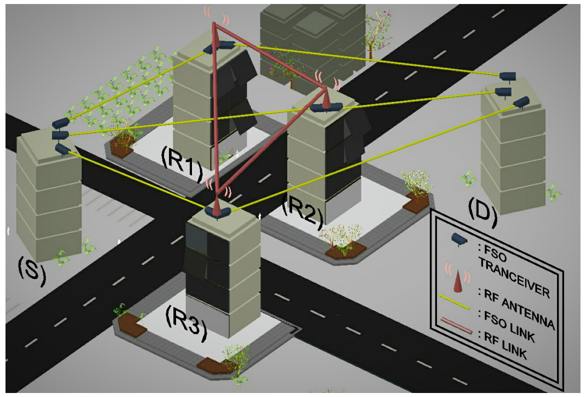

Fig. 1. A cooperative FSO network with three RF inter-connected relays.

along the R-R links, only a fraction of these packets are shared in our case. This is motivated by the fact that the FSO and RF systems operate at different data rates. Finally, an additional contribution of the paper resides in applying the cut-set method for deriving a tight upper-bound on the network outage probability.

2. System model

2.1. Cooperation strategy

Consider a cooperative FSO system where N relays denoted by $R_1, \ldots, R_N$ assist the communications between a source node S and a destination node D. As in conventional cooperative FSO systems, FSO links are assumed to be established between S and D, between S and the relays as well as between the relays and D. In addition to these $2N + 1$ FSO links, the proposed system takes advantage from the presence of the backup RF links for the sake of establishing RF communications between the relays. A cooperative FSO network with RF inter-connected relays is shown in Fig. 1 in the case of $N = 3$. In this context, a message broadcasted by the RF antenna of a certain relay will be overheard by all remaining relays. This results in establishing additional $\binom{N}{2}$ RF links between the relays that can be further exploited to enhance the system performance. Finally, the communication between any two nodes in the network involves the ACK/NACK mechanism where the receiver informs the transmitter about the packet's reception status.

The proposed scheme can be perceived as an opportunistic protocol that can be implemented as follows.

– *S-D phase:* S transmits an information packet to D along the direct S-D FSO link.

– *S-R phase:* In case of a positive acknowledgement (ACK), S proceeds to the transmission of a new packet to D. Otherwise, S transmits the packet to the relays along the N FSO links $S\text{-}R_1, \ldots, S\text{-}R_N$.

– *R-R phase:* All relays that have successfully decoded the packet proceed to broadcasting this packet along the RF links. In order to avoid interference, it is assumed that different RF frequency bands are allocated to each one of the relays.

– *R-D phase:* All relays that successfully received the packet proceed to retransmitting this packet along the corresponding R-D links. In this context, the involved relays could have received the packet either from S (along the FSO link) or from another relay (along the RF links).

It is extremely important to highlight that the proposed scheme can be implemented despite the fact that the FSO links and RF links operate at different data rates as is explained in what follows. Assume that the data rate of the FSO links is M times larger than the data rate of the RF links implying that the R-R phase is M times slower than the remaining transmission phases.

In other words, in the time needed to share a single packet among the relays along the R-R RF links, S would have proceeded to the transmission of M packets along the S-D FSO link. On the other hand, denote by P_e the average packet error rate along the direct S-D link. In this context, two packets with erroneous detection (resulting in a NACK) will be separated by $1/P_e$ packet durations on average.

Therefore, the proposed strategy can be put in place if $1/P_e > M$. Typical values of M range between 10 and 100 [1, 2, 4] where, for example, in [4] a 1 Gbps FSO link was combined with a 96 Mbps WiMAX RF link in the context of hybrid RF/FSO communications. In this context, while the bandwidth of RF links is smaller, the data rate of these links is positively impacted by the fact that higher order modulations are often applied compared to the binary On-Off-Keying (OOK) modulation that is often used with FSO links. Consequently, P_e must be roughly smaller than 10^{-2} that does not constitute a limiting constraint since FSO systems are designed to operate at low error rates (typically below 10^{-6}). In other words, the relation $MP_e < 1$ can be easily satisfied in practical scenarios where the signal-to-noise ratio (SNR) is not very low. This renders the proposed scheme suitable for the average-to-high SNR range which does not constitute an exception to the conventional cooperative-diversity fading-mitigation techniques that are typically designed to achieve the highest performance gains at high SNRs.

While $1/P_e$ corresponds to an average value, additional packets might still be erroneously detected (along the S-D link) in the time span of M packet durations even though this event occurs with low probability. A practical solution that avoids dropping such packets consists of equipping S with a buffer (data queue) where the relation $MP_e \ll 1$ ensures that very small buffer sizes are sufficient in practice.

2.2. FSO S-D, S-R and R-D links

2.2.1. Channel model

In what follows, S and D will be denoted by R_0 and R_{N+1}, respectively. Denote by $I_{i,j}$ the irradiance along the FSO link R_i-R_j. In this work, we adopt a channel model that takes into account the combined effects of path loss, atmospheric turbulence-induced scintillation and misalignment-induced fading caused by pointing errors. The probability density function (pdf) of $I_{i,j}$ was derived in [12] assuming a gamma-gamma turbulence model and a Gaussian spatial intensity profile falling on a circular aperture at the receiver:

$$f_{I_{i,j}}(I) = \frac{\alpha_{i,j}\beta_{i,j}\xi_{i,j}^2}{A_{i,j}I_{i,j}^{(l)}\Gamma(\alpha_{i,j})\Gamma(\beta_{i,j})} G_{1,3}^{3,0}\left[\frac{\alpha_{i,j}\beta_{i,j}}{A_{i,j}I_{i,j}^{(l)}}I \middle| \begin{matrix} \xi_{i,j}^2 \\ \xi_{i,j}^2-1, \alpha_{i,j}-1, \beta_{i,j}-1 \end{matrix}\right] \tag{1}$$

where $\Gamma(.)$ is the Gamma function and $G_{p,q}^{m,n}[.]$ is the Meijer G-function. In (1), $\alpha_{i,j}$ and $\beta_{i,j}$ stand for the distance-dependent parameters of the gamma-gamma distribution:

$$\alpha_{i,j} = \left[\exp\left(0.49\sigma_{R,i,j}^2/(1+1.11\sigma_{R,i,j}^{12/5})^{7/6}\right)-1\right]^{-1} \tag{2}$$

$$\beta_{i,j} = \left[\exp\left(0.51\sigma_{R,i,j}^2/(1+0.69\sigma_{R,i,j}^{12/5})^{5/6}\right)-1\right]^{-1} \tag{3}$$

where $\sigma_{R,i,j}^2 = 1.23C_n^2k^{7/6}d_{i,j}^{11/6}$ is the Rytov variance where $d_{i,j}$ stands for the length of the link R_i-R_j, k is the wave number and C_n^2 denotes the refractive index structure parameter.

In (1), the parameters $A_{i,j}$ and $\xi_{i,j}$ are related to the pointing errors. $A_{i,j}$ is given by $A_{i,j} = [\mathrm{erf}(v_{i,j})]^2$ where erf(.) stands for the error function with $v_{i,j} = \sqrt{\pi/2}(a_{i,j}/\omega_{z,i,j})$ where $a_{i,j}$ is the radius of the receiver and $\omega_{z,i,j}$ is the beam waist along the link R_i-R_j. $\xi_{i,j} = \omega_{z_{eq},i,j}/2\sigma_{s,i,j}$ where $\sigma_{s,i,j}$ stands for the pointing error displacement standard deviation at the receiver and $\omega^2_{z_{eq},i,j} = \omega^2_{z,i,j}\sqrt{\pi}\mathrm{erf}(v_{i,j})/[2v_{i,j}e^{-v^2_{i,j}}]$. Finally, the atmospheric loss is given by $I^{(l)}_{i,j} = e^{-\sigma d_{i,j}}$ where σ is the attenuation coefficient.

2.2.2. Outage probability

Consider a non-coherent FSO system based on Intensity-Modulation and Direct-Detection (IM/DD). The system outage probability will be reported in terms of the optical power margin $\mathcal{P}_M = \sqrt{\frac{\overline{\gamma}_{0,N+1}}{\gamma_{th}}}$ where $\overline{\gamma}_{0,N+1}$ stands for the average electrical SNR along the direct link S-D while γ_{th} denotes the threshold SNR below which an outage event occurs.

The instantaneous electrical SNR $\gamma_{i,j}$ along the link R$_i$-R$_j$ is related to the average electrical SNR $\overline{\gamma}_{0,N+1}$ by the following relation [11]:

$$\gamma_{i,j} = \overline{\gamma}_{0,N+1}\left(\frac{\xi^2_{0,N+1}+1}{A_{0,N+1}I^{(l)}_{0,N+1}\xi^2_{0,N+1}}\right)^2\left(\frac{I_{i,j}}{N_{\text{link}}}\right)^2 \tag{4}$$

where $N_{\text{link}} = 2N+1$ stands for the total number of optical links in the FSO network.

Based on (4), the outage probability $p_{i,j} \triangleq \Pr\left(\gamma_{i,j} < \gamma_{th}\right)$ along the link R$_i$-R$_j$ can be written as:

$$p_{i,j} = F_{I_{i,j}}\left(\frac{N_{\text{link}}}{\mathcal{P}_M}\frac{A_{0,N+1}I^{(l)}_{0,N+1}\xi^2_{0,N+1}}{\xi^2_{0,N+1}+1}\right) \tag{5}$$

where $F_{I_{i,j}}(.)$ stands for the cumulative distribution function (cdf) associated with the pdf in (1) [12]:

$$F_{I_{i,j}}(I) = \frac{\xi^2_{i,j}}{\Gamma(\alpha_{i,j})\Gamma(\beta_{i,j})}G^{3,1}_{2,4}\left[\frac{\alpha_{i,j}\beta_{i,j}}{A_{i,j}I^{(l)}_{i,j}}I\left|\begin{matrix}1,\xi^2_{i,j}+1\\ \xi^2_{i,j},\alpha_{i,j},\beta_{i,j},0\end{matrix}\right.\right] \tag{6}$$

2.3. RF R-R links

The RF links between the relays are assumed to follow either the Rician or the Rayleigh fading models depending on the presence or absence of a direct LOS between the communicating nodes, respectively. In what follows, $\overline{\Omega}_{i,j}$ stands for the average SNR along the RF link R$_i$-R$_j$ while $\Omega_{th} = 2^{2R}-1$ stands for the threshold SNR where R is the number of bits transmitted per channel use.

For Rayleigh fading, the outage probability along the RF link R$_i$-R$_j$ can be written as:

$$p_{i,j} = 1-\exp\left(-\frac{\Omega_{th}}{\overline{\Omega}_{i,j}}\right) \tag{7}$$

In the case of Rician fading [13]:

$$p_{i,j} = 1-Q_1\left(\sqrt{2K_{i,j}},\sqrt{\frac{2(K_{i,j}+1)\Omega_{th}}{\overline{\Omega}_{i,j}}}\right) \tag{8}$$

where $Q_1(.,.)$ stands for the first-order Marcum Q function. $K_{i,j}$ is the Rician factor defined as the ratio of the power in the LOS component to the power in the non-LOS components.

Finally, following from the reciprocity of the RF channels, the relation $p_{i,j} = p_{j,i}$ holds for any pair of relay nodes.

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3. Performance analysis

3.1. Exact system outage probability

The outage probability of the network can be separated into two terms as follows:

$$P_{\text{Net}} = p_{0,N+1} P_{\text{out}} \tag{9}$$

where, evidently, the network will not suffer from outage if the direct link S-D (or, equivalently, R_0-R_{N+1}) is not in outage.

The probability P_{out} (related to the indirect links) will be evaluated based on the conditional probability method where the system will be reduced into simpler subsystems comprising simple series or parallel connections. The system of indirect S-R, R-R and R-D links will be referred to as "*system*" in what follows. Links in series will not suffer from outage only if all constituents sub-links are not in outage while links in parallel will suffer from outage only if all constituents sub-links are in outage:

$$\left(1 - P_{(\text{R}_i-\text{R}_j)\rightleftharpoons(\text{R}_j-\text{R}_k)\rightleftharpoons\cdots}\right) = (1-p_{i,j})(1-p_{j,k})\cdots \tag{10}$$

$$P_{(\text{R}_i-\text{R}_j)\shortparallel(\text{R}_i-\text{R}_k)\shortparallel\cdots} = p_{i,j}p_{i,k}\cdots \tag{11}$$

where $\rightleftharpoons$ and $\shortparallel$ stand for the series and parallel connections, respectively. In (10)-(11), $P_{\dots}$ stands for the outage probability of the equivalent series or parallel subsystem.

In what follows, conditioning will be performed on the status of the RF inter-relay links. As illustrative examples, we first consider the cases $N = 2$ and $N = 3$ before tackling the general case of systems with an arbitrary number of relays.

3.1.1. $N = 2$ relays

Two cases will arise. (i): The inter-relay link is in outage with probability $p_{1,2}$ (that is derived according to (7) or (8)). In this case, the system reduces to the parallel connection between two constituent subsystems as follows: $[(\text{R}_0\text{-}\text{R}_1)\rightleftharpoons(\text{R}_1\text{-}\text{R}_3)]\shortparallel[(\text{R}_0\text{-}\text{R}_2)\rightleftharpoons(\text{R}_2\text{-}\text{R}_3)]$. Invoking (11) followed by (10) results in the following expression of the system outage probability:

$$P_{\text{out}}^{(1)} = [1-(1-p_{0,1})(1-p_{1,3})][1-(1-p_{0,2})(1-p_{2,3})] \tag{12}$$

(ii): The inter-relay link is not in outage with probability $1 - p_{1,2}$ implying that the two relays are interconnected by the RF link. In other words, R_1 and R_2 can be combined into a single node and the system reduces to the series connection between two parallel subsystems: $[(\text{R}_0\text{-}\text{R}_1)\shortparallel(\text{R}_0\text{-}\text{R}_2)]\rightleftharpoons[(\text{R}_1\text{-}\text{R}_3)\shortparallel(\text{R}_2\text{-}\text{R}_3)]$. Invoking (10) followed by (11), the system outage probability can be written as:

$$P_{\text{out}}^{(2)} = 1-(1-p_{0,1}p_{0,2})(1-p_{1,3}p_{2,3}) \tag{13}$$

Combining (12) and (13) results in $P_{\text{out}} = p_{1,2}P_{\text{out}}^{(1)} + (1-p_{1,2})P_{\text{out}}^{(2)}$.

3.1.2. More convenient representation

The reduced systems' outage probabilities $P_{\text{out}}^{(1)}$ and $P_{\text{out}}^{(2)}$ (and all subsequent similar probabilities) can be written in a more convenient and unified manner as follows. Let $\mathcal{S}$ be a subset of $\{1,\dots,N\}$ containing the indices of the relays that are successfully (with no outage) interconnected via the RF links. In other words, the availability of the information packet at any relay in $\mathcal{S}$ will imply the availability at all remaining relays of this set. In this case, relays $\text{R}_{\mathcal{S}_1}$, $\text{R}_{\mathcal{S}_2}$, ..., $\text{R}_{\mathcal{S}_{|\mathcal{S}|}}$ can be combined into a single node (where $\mathcal{S}_n$ denotes the n-th element of $\mathcal{S}$ with $|\mathcal{S}|$ standing for the cardinality of $\mathcal{S}$). Therefore, relays in $\mathcal{S}$ will form a reduced system that corresponds to the series connection of two parallel subsystems as follows:

$$\left[(\text{R}_0-\text{R}_{\mathcal{S}_1})\shortparallel\cdots\shortparallel(\text{R}_0-\text{R}_{\mathcal{S}_{|\mathcal{S}|}})\right] \rightleftharpoons \left[(\text{R}_{\mathcal{S}_1}-\text{R}_{N+1})\shortparallel\cdots\shortparallel(\text{R}_{\mathcal{S}_{|\mathcal{S}|}}-\text{R}_{N+1})\right] \tag{14}$$

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The reduced system in (14) will be parallel to other similar reduced systems corresponding to other disjoint subsets of $\{1,\ldots,N\}$. Consequently, the contribution of $\mathcal{S}$ to the system outage probability can be captured by the following probability:

$$Q_{\mathcal{S}} \triangleq 1 - \left[1 - \prod_{n\in\mathcal{S}} p_{0,n}\right]\left[1 - \prod_{n\in\mathcal{S}} p_{n,N+1}\right] \tag{15}$$

where (10) was invoked followed by (11).

Based on the notation in (15), the probabilities in (12) and (13) can be written as $P_{\text{out}}^{(1)} = Q_{\{1\}}Q_{\{2\}}$ and $P_{\text{out}}^{(2)} = Q_{\{1,2\}}$, respectively.

3.1.3. $N = 3$ relays

Conditioning on the three RF inter-relay links R_1-R_2, R_1-R_3 and R_2-R_3, the system outage probability can be expanded as follows:

$$\begin{aligned} P_{\text{out}} = {} & p_{1,2}p_{1,3}p_{2,3}P_{\text{out}}^{(1)} + (1-p_{1,2})p_{1,3}p_{2,3}P_{\text{out}}^{(2)} + p_{1,2}(1-p_{1,3})p_{2,3}P_{\text{out}}^{(3)} \\ & + p_{1,2}p_{1,3}(1-p_{2,3})P_{\text{out}}^{(4)} + (1-p_{1,2})(1-p_{1,3})p_{2,3}P_{\text{out}}^{(5)} + (1-p_{1,2})p_{1,3}(1-p_{2,3})P_{\text{out}}^{(6)} \\ & + p_{1,2}(1-p_{1,3})(1-p_{2,3})P_{\text{out}}^{(7)} + (1-p_{1,2})(1-p_{1,3})(1-p_{2,3})P_{\text{out}}^{(8)} \end{aligned} \tag{16}$$

(i) $P_{\text{out}}^{(1)}$: In this case, all inter-relay links are in outage (the system reduces to the conventional parallel-relaying system). Consequently, the system will be in outage if the three parallel links R_0-R_1-R_4, R_0-R_2-R_4 and R_0-R_3-R_4 are in outage implying that $P_{\text{out}}^{(1)} = Q_{\{1\}}Q_{\{2\}}Q_{\{3\}}$ based on (15). (ii) $P_{\text{out}}^{(2)}$: In this case, R_1 and R_2 can be combined into a single node and the system can be broken down into two parallel subsystems: $\{[(R_0\text{-}R_1)\parallel(R_0\text{-}R_2)]\rightleftharpoons[(R_1\text{-}R_4)\parallel(R_2\text{-}R_4)]\}\parallel[(R_0\text{-}R_3)\rightleftharpoons(R_3\text{-}R_4)]$ implying that $P_{\text{out}}^{(2)} = Q_{\{1,2\}}Q_{\{3\}}$. Similarly, $P_{\text{out}}^{(3)} = Q_{\{1,3\}}Q_{\{2\}}$ and $P_{\text{out}}^{(4)} = Q_{\{2,3\}}Q_{\{1\}}$. (iii) $P_{\text{out}}^{(5)}$: In this case, the links R_1-R_2 and R_1-R_3 are not in outage. In other words, if an information packet is successfully received at R_1 from S, then this packet will be successfully forwarded to R_2 and R_3 along the links R_1-R_2 and R_1-R_3. On the other hand, if the packet is available at R_2, then it will be successfully forwarded to R_1 and R_3 along the links R_2-R_1 and subsequently R_1-R_3. Finally, a packet available at R_3 will be available to all other relays through the links R_3-R_1 and R_1-R_2. Therefore, all relays can be combined into a single node implying that $P_{\text{out}}^{(5)} = Q_{\{1,2,3\}}$. Similarly, $P_{\text{out}}^{(6)} = P_{\text{out}}^{(7)} = P_{\text{out}}^{(8)} = Q_{\{1,2,3\}}$ since, in all of the corresponding cases, all of the relays are interconnected and, hence, can be combined into a single node.

3.1.4. Any number of relays

The N relays are interconnected by the $\binom{N}{2}$ RF links R_n-$R_{n'}$ for $(n,n') \in \{1,\ldots,N\}^2$ with $n < n'$. We define the $\binom{N}{2}$-dimensional vector $\mathbf{F}$ as follows:

$$\mathbf{F} \triangleq \left[f_{1,2},\ldots,f_{1,N},f_{2,3},\ldots,f_{2,N},\ldots,f_{N-1,N}\right] \tag{17}$$

where $f_{n,n'}$ indicates the status of the RF inter-relay link R_n-$R_{n'}$ with:

$$f_{n,n'} = \begin{cases} 0, & \text{link } R_n\text{-}R_{n'} \text{ in outage;} \\ 1, & \text{link } R_n\text{-}R_{n'} \text{ not in outage.} \end{cases} \tag{18}$$

Based on the above notation, the system outage probability can be expanded as follows:

$$P_{\text{out}} = \sum_{\mathbf{F}\in\{0,1\}^{\binom{N}{2}}} \pi(\mathbf{F}) \prod_{m=1}^{M} Q_{\mathcal{S}_m^{(\mathbf{F})}} \tag{19}$$

where the probability $Q_{\mathcal{S}_m^{(\mathbf{F})}}$ is given in (15) while $\pi(\mathbf{F}) = \left[\pi(f_{1,2}), \ldots, \pi(f_{1,N}), \ldots, \pi(f_{N-1,N})\right]$ with:

$$\pi(f_{n,n'}) = \begin{cases} p_{n,n'}, & f_{n,n'} = 0; \\ 1 - p_{n,n'}, & f_{n,n'} = 1. \end{cases} \tag{20}$$

where $p_{n,n'}$ is the outage probability of the RF link R_n-$\mathrm{R}_{n'}$ that can be determined from (7) or (8).

In (19), the sets $\mathcal{S}_1^{(F)}, \ldots, \mathcal{S}_M^{(F)}$ correspond to a partitioning of the set $\{1, \ldots, N\}$. This partitioning depends on the value of the vector $\mathbf{F}$ and is determined such that:

$$\forall\ (n, n') \in \mathcal{S}_m^{(F)} \times \mathcal{S}_{m'}^{(F)} \text{ (with } m' \neq m) : f_{n,n'} = 0 \tag{21}$$

implying that all RF links connecting elements of $\mathcal{S}_m^{(F)}$ to elements of $\mathcal{S}_{m'}^{(F)}$ are in outage. In an equivalent way, elements n and n' of the same subset $\mathcal{S}_m^{(F)}$ satisfy the following relation:

$$\forall\ (n, n') \in \mathcal{S}_m^{(F)} \times \mathcal{S}_m^{(F)} \text{ (with } n' \neq n) : \exists\, \mathcal{K} \subset \mathcal{S}_m^{(F)} \mid f_{n,\mathcal{K}_1} = f_{\mathcal{K}_1,\mathcal{K}_2} = \cdots = f_{\mathcal{K}_{|\mathcal{K}|},n'} = 1 \tag{22}$$

implying that R_n and $\mathrm{R}_{n'}$ are successfully connected through the $|\mathcal{K}|$ intermediate relays $\mathrm{R}_{\mathcal{K}_1}, \ldots, \mathrm{R}_{\mathcal{K}_{|\mathcal{K}|}}$ where $\mathcal{K}_i$ is the i-th element of $\mathcal{K}$.

Based on the above partitioning, all relays whose indices fall in $\mathcal{S}_m^{(F)}$ can be combined into a single node resulting in the outage probability $Q_{\mathcal{S}_m^{(F)}}$. In this context, depending on the specific value of $\mathbf{F}$, the system can be reduced into M parallel systems whose outage probabilities can be multiplied.

Example-1: For evaluating $P_{\text{out}}^{(4)}$ in (16), $\mathbf{F} = [0, 0, 1]$ implying that the set $\{1, 2, 3\}$ is partitioned as $\{1\} \cup \{2, 3\}$ in this case since $f_{1,2} = f_{1,3} = 0$.

Example-2: Consider the case $N = 8$ where the nonzero elements of $\mathbf{F}$ are given by: $f_{1,2} = f_{1,3} = f_{2,5} = f_{4,6} = 1$. In this case, the set $\{1, \ldots, 8\}$ must be partitioned as $\mathcal{S}_1^{(\mathbf{F})} = \{1, 2, 3, 5\}$, $\mathcal{S}_2^{(\mathbf{F})} = \{4, 6\}$, $\mathcal{S}_3^{(\mathbf{F})} = \{7\}$ and $\mathcal{S}_4^{(\mathbf{F})} = \{8\}$. Consider the two elements (n, n') of $\mathcal{S}_1^{(\mathbf{F})}$. (i): For $(n, n') = (1, 2)$, $f_{1,2} = 1$ implying that the link R_1-R_2 is not in outage and that these relays are directly connected. The same holds for $(n, n') = (1, 3)$ and $(n, n') = (2, 5)$. (ii): For $(n, n') = (1, 5)$, $f_{1,2} = f_{2,5} = 1$ implying that R_1 and R_5 are connected through R_2 even though the link R_1-R_5 is in outage ($f_{1,5} = 0$). The same holds for $(n, n') = (2, 3)$ where R_2 and R_3 are connected though R_1: $f_{2,1} = f_{1,3} = 1$ (note that $f_{i,j} = f_{j,i}$ following from the reciprocity of the channel). (iii): For $(n, n') = (3, 5)$, $f_{3,1} = f_{1,2} = f_{2,5} = 1$ implying that R_3 and R_5 are successfully connected to each other through the pair of relays R_1 and R_2.

Example-3: If $\mathbf{F}$ contains more than $\binom{N-1}{2}$ nonzero elements, then all relays can be combined in one node implying that the last term in (19) simplifies to $Q_{\{1,\ldots,N\}}$. For example, for $N = 3$, $P_{\text{out}}^{(5)} = P_{\text{out}}^{(6)} = P_{\text{out}}^{(7)} = Q_{\{1,2,3\}}$ in (16) since the corresponding $\mathbf{F}$ vectors contain 2 nonzero elements each while $P_{\text{out}}^{(8)} = Q_{\{1,2,3\}}$ since the corresponding $\mathbf{F}$ vector contains 3 nonzero elements.

3.2. Upper-bound on the system outage probability

While (19) provides the exact system outage probability, it suffers from the limitation of expressing the outage probability as the summation of $2^{\binom{N}{2}}$ terms. In this context, the number of summands increases very rapidly with N resulting in intractable outage probability expressions. Moreover, these summands can not be evaluated in a straightforward manner since they involve the partitioning of the N relays. This motivates providing a simpler expression that serves as an upper-bound to P_{out}. As will be explained later in Section 3.3, the proposed bound will be particularly useful for carrying out an asymptotic analysis and evaluating the diversity gain.

One of the most popular methods for the reliability evaluation of complex systems is the cut-set method [14]. In reliability theory, a cut-set is defined as *"the set of components whose failure alone will cause the system failure"* [14]. For the communication problem at hand,

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a cut-set can be equivalently defined as the set of links whose outage alone will cause the system outage. A cut-set is said to be minimal if it does not contain any subset of links whose outage alone will cause the system outage. Once the minimal cut-sets denoted by $C_1, \ldots, C_m$ are identified, the union bound can be invoked and the system outage probability $P_{\text{out}} = \Pr\left((C_1 \text{ in outage}) \cup \cdots \cup (C_m \text{ in outage})\right)$ can be upper-bounded as:

$$P_{\text{out}} \leq P_{\text{U.B.}} \triangleq \sum_{i=1}^{m} \Pr\left(C_i \text{ in outage}\right) \tag{23}$$

For simple systems, a visual inspection is sufficient for identifying the minimal cut-sets. For example, for $N = 2$, the minimal cut-sets are given by $\{R_0\text{-}R_1, R_0\text{-}R_2\}$, $\{R_1\text{-}R_3, R_2\text{-}R_3\}$, $\{R_0\text{-}R_1, R_1\text{-}R_2, R_2\text{-}R_3\}$ and $\{R_0\text{-}R_2, R_1\text{-}R_2, R_1\text{-}R_3\}$ implying that:

$$P_{\text{U.B.}} = p_{0,1}p_{0,2} + p_{1,3}p_{2,3} + p_{0,1}p_{1,2}p_{2,3} + p_{0,2}p_{1,2}p_{1,3} \tag{24}$$

For complex systems (in our case, networks with large number of relays), several algorithms exist for computing the minimal cut-sets; the most famous among these is probably the MOCUS algorithm [15]. An efficient implementation of MOCUS that is based on data-structures was discussed in [16].

Based on the cut-set algorithm, the sought upper-bound can be expressed as:

$$P_{\text{U.B.}} = \sum_{n=0}^{N} \sum_{i=1}^{\binom{N}{n}} \left[\prod_{j \in \mathcal{I}_{n,i}} p_{j,N+1} \right] \left[\prod_{j' \in \overline{\mathcal{I}}_{n,i}} p_{0,j'} \right] \left[\prod_{j \in \mathcal{I}_{n,i}\ ;\ j' \in \overline{\mathcal{I}}_{n,i}} p_{j,j'} \right] \tag{25}$$

where $\mathcal{I}_{n,1}, \ldots, \mathcal{I}_{n,\binom{N}{n}}$ are all possible subsets of $\{1, \ldots, N\}$ having n elements each while $\overline{\mathcal{I}}_{n,i} = \{1, \ldots, N\} \backslash \mathcal{I}_{n,i}$.

The expression in (25) originates from identifying the 2^N possible minimal cut-sets as follows. Consider the minimal cut-set comprising the links $\{R_j\text{-}D\}_{j \in \mathcal{I}_{n,i}}$ for a certain subset $\mathcal{I}_{n,i}$ of the relays. Evidently, being elements of a cut-set, these links must be in outage resulting in the first term in (25). Now since the links $\{R_{j'}\text{-}D\}_{j' \in \overline{\mathcal{I}}_{n,i}}$ are not in outage (by construction), then the corresponding S-R links must in outage; otherwise, the information packet can be delivered along the corresponding two-hop S-R-D optical link resulting in no outage. Therefore, the links $\{S\text{-}R_{j'}\}_{j' \in \overline{\mathcal{I}}_{n,i}}$ must incontestably be included in the cut-set resulting in the second term in (25). Similarly, since the links $\{R_{j'}\text{-}D\}_{j' \in \overline{\mathcal{I}}_{n,i}}$ are not in outage, then the corresponding relays must not be able to successfully receive the packet from any other relay (along the RF links); otherwise, this packet will eventually reach D along the R-D optical link. Following from the failure of the links $\{S\text{-}R_{j'}\}_{j' \in \overline{\mathcal{I}}_{n,i}}$ (second term in (25)) the packet can reach $R_{j'}$ (for $j' \in \overline{\mathcal{I}}_{n,i}$) only from the relay R_j for $j \in \mathcal{I}_{n,i}$. Consequently, the links $\{R_j\text{-}R_{j'}\}_{j \in \mathcal{I}_{n,i}; j' \in \overline{\mathcal{I}}_{n,i}}$ must be included in the cut-set resulting in the last term in (25).

Finally, the outage probability of the conventional parallel-relaying systems can be obtained by setting the third term in (25) to 1. In other words, this third term results from implementing the inter-relay cooperation along the RF links. Given that this term is the product of some outage probabilities and is hence less than or equal to 1, then it contributes to decreasing the system outage probability.

For example, for $N = 4$, while the exact outage probability in (19) comprises $2^{\binom{N}{2}} = 64$ terms, the upper-bound in (25) comprises the $2^N = 16$ terms having the lowest orders and can be readily written as:

$$P_{\text{U.B.}} = P_{\text{U.B.}}^{(0)} + P_{\text{U.B.}}^{(1)} + P_{\text{U.B.}}^{(2)} + P_{\text{U.B.}}^{(3)} + P_{\text{U.B.}}^{(4)} \tag{26}$$

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where $P^{(0)}_{\text{U.B.}} = p_{0,1}p_{0,2}p_{0,3}p_{0,4}$ and $P^{(4)}_{\text{U.B.}} = p_{1,5}p_{2,5}p_{3,5}p_{4,5}$ while $P^{(1)}_{\text{U.B.}}$, $P^{(2)}_{\text{U.B.}}$ and $P^{(3)}_{\text{U.B.}}$ take the following expressions:

$$\begin{aligned} P^{(1)}_{\text{U.B.}} &= p_{1,5}(p_{0,2}p_{0,3}p_{0,4})(p_{1,2}p_{1,3}p_{1,4}) + p_{2,5}(p_{0,1}p_{0,3}p_{0,4})(p_{2,1}p_{2,3}p_{2,4}) \\ &\quad + p_{3,5}(p_{0,1}p_{0,2}p_{0,4})(p_{3,1}p_{3,2}p_{3,4}) + p_{4,5}(p_{0,1}p_{0,2}p_{0,3})(p_{4,1}p_{4,2}p_{4,3}) \end{aligned} \tag{27}$$

$$\begin{aligned} P^{(2)}_{\text{U.B.}} &= (p_{1,5}p_{2,5})(p_{0,3}p_{0,4})(p_{1,3}p_{1,4}p_{2,3}p_{2,4}) + (p_{1,5}p_{3,5})(p_{0,2}p_{0,4})(p_{1,2}p_{1,4}p_{3,2}p_{3,4}) \\ &\quad + (p_{1,5}p_{4,5})(p_{0,2}p_{0,3})(p_{1,2}p_{1,3}p_{4,2}p_{4,3}) + (p_{2,5}p_{3,5})(p_{0,1}p_{0,4})(p_{2,1}p_{2,4}p_{3,1}p_{3,4}) \\ &\quad + (p_{2,5}p_{4,5})(p_{0,1}p_{0,3})(p_{2,1}p_{2,3}p_{4,1}p_{4,3}) + (p_{3,5}p_{4,5})(p_{0,1}p_{0,2})(p_{3,1}p_{3,2}p_{4,1}p_{4,2}) \end{aligned} \tag{28}$$

$$\begin{aligned} P^{(3)}_{\text{U.B.}} &= (p_{1,5}p_{2,5}p_{3,5})p_{0,4}(p_{1,4}p_{2,4}p_{3,4}) + (p_{1,5}p_{2,5}p_{4,5})p_{0,3}(p_{1,3}p_{2,3}p_{4,3}) \\ &\quad + (p_{1,5}p_{3,5}p_{4,5})p_{0,2}(p_{1,2}p_{3,2}p_{4,2}) + (p_{2,5}p_{3,5}p_{4,5})p_{0,1}(p_{2,1}p_{3,1}p_{4,1}) \end{aligned} \tag{29}$$

3.3. Asymptotic analysis and diversity gain

From (25), the cardinalities of the sets $\mathcal{I}_{n,i}$ and $\overline{\mathcal{I}}_{n,i}$ are equal to n and $N-n$, respectively. Therefore, the summands in (25) correspond to the product of $N + n(N-n)$ outage probability terms for the values of n ranging between 0 and N. Moreover, N of these terms correspond to the S-R and R-D optical links while the remaining $n(N-n)$ terms correspond to the R-R RF links.

Therefore, the dominant terms in the summation correspond to $n = 0$ and $n = N$ comprising the product of N outage probabilities each. For $n = 0$, the set $\mathcal{I}_{n,i} = \phi$ (the empty set) implying, from (25), that the corresponding probability is $\prod_{n=1}^{N} p_{0,n}$ (since $\overline{\mathcal{I}}_{n,i} = \{1,\ldots,N\}$ in this case). On the other hand, for $n = N$, $\mathcal{I}_{n,i} = \{1,\ldots,N\}$ implying that the corresponding probability is $\prod_{n=1}^{N} p_{n,N+1}$ (since $\overline{\mathcal{I}}_{n,i} = \phi$ in this case).

On the other hand, the remaining terms corresponding to $n \notin \{0, N\}$ contain at least $N-1$ additional multiplicands that originate from the last product in (25). These multiplicands comprise the outage probabilities along the inter-relay RF links and result from the implementation of the proposed cooperation strategy. For large values of the SNR along the FSO and RF links, all of these additional terms can be ignored implying that the upper-bound in (25) can be approximated by $\prod_{n=1}^{N} p_{0,n} + \prod_{n=1}^{N} p_{n,N+1}$ which, from (9), results in:

$$P_{\text{Net}} \approx p_{0,N+1}\left[\prod_{n=1}^{N} p_{0,n} + \prod_{n=1}^{N} p_{n,N+1}\right] \tag{30}$$

for large values of $\mathcal{P}_M$ in (5) and of $\{\overline{\Omega}_{i,j}\}_{\substack{i,j=1\\ i<j}}^{N}$ in (7)-(8).

In the absence of inter-relay cooperation, the last product in (25) is always equal to 1 implying that each summand comprises exactly N outage probability terms and, hence, non of these summands can be ignored. Following from [10], (25) can be factorized as:

$$P^{(0)}_{\text{Net}} \approx p_{0,N+1}\prod_{n=1}^{N}\left[p_{0,n} + p_{n,N+1}\right] \tag{31}$$

where the superscript 0 refers to the conventional parallel-relaying scheme.

From [11], the FSO outage probability in (5) behaves asymptotically as $\mathcal{P}_M^{-\zeta_{i,j}}$ showing that the diversity order along the FSO link R_i-R_j is equal to $\zeta_{i,j} = \min\{\beta_{i,j}, \xi^2_{i,j}\}$ where $\beta_{i,j}$ and $\xi_{i,j}$ are the parameters of the pdf in (1) that are related to the scintillation and pointing errors, respectively. Therefore, from (30), the diversity order that can be achieved by the proposed strategy is given by:

$$\zeta_{\text{Net}} = \zeta_{0,N+1} + \min\left\{\sum_{n=1}^{N}\zeta_{0,n}, \sum_{n=1}^{N}\zeta_{n,N+1}\right\} \tag{32}$$

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while, from (31), the diversity order of the parallel-relaying scheme is:

$$\zeta_{\text{Net}}^{(0)} = \zeta_{0,N+1} + \sum_{n=1}^{N} \min\left\{\zeta_{0,n}, \zeta_{n,N+1}\right\} \tag{33}$$

Following from the fact that $\zeta_{0,n} \geq \min\left\{\zeta_{0,n}, \zeta_{n,N+1}\right\}$ and $\zeta_{n,N+1} \geq \min\left\{\zeta_{0,n}, \zeta_{n,N+1}\right\}$, then $\sum_{n=1}^{N} \zeta_{0,n} \geq \sum_{n=1}^{N} \min\left\{\zeta_{0,n}, \zeta_{n,N+1}\right\}$ and $\sum_{n=1}^{N} \zeta_{n,N+1} \geq \sum_{n=1}^{N} \min\left\{\zeta_{0,n}, \zeta_{n,N+1}\right\}$ implying that $\zeta_{\text{Net}} \geq \zeta_{\text{Net}}^{(0)}$. This highlights the capability of the proposed scheme in enhancing the network diversity order.

In this context, it is worth noting that no diversity gains are obtained only under the two following scenarios. (i): $\min\left\{\zeta_{0,n}, \zeta_{n,N+1}\right\} = \zeta_{0,n}$ for $n = 1, \ldots, N$ implying that all relays are closer to D and/or the pointing errors along all R-D links are small. In this case, $\zeta_{\text{Net}} = \zeta_{\text{Net}}^{(0)} = \zeta_{0,N+1} + \sum_{n=1}^{N} \zeta_{0,n}$. (ii): $\min\left\{\zeta_{0,n}, \zeta_{n,N+1}\right\} = \zeta_{n,N+1}$ for $n = 1, \ldots, N$ implying that all relays are closer to S and/or the pointing errors along all S-R links are small. In this case, $\zeta_{\text{Net}} = \zeta_{\text{Net}}^{(0)} = \zeta_{0,N+1} + \sum_{n=1}^{N} \zeta_{n,N+1}$.

4. Numerical results

The refractive index structure constant and the attenuation constant are set to $C_n^2 = 1\times10^{-14}$ $\text{m}^{-2/3}$ and $\sigma = 0.44$ dB/km. We also fix $\lambda = 1550$ nm and $R = 1$ bit per channel use (bpcu). In all scenarios, the distance between S and D is $d_{0,N+1} = 3$ km. We assume that the SNRs along the RF links are the same: $\overline{\Omega}_{i,j} \triangleq \Omega$ for all $i, j \in \{1, \ldots, N\}$ with $i < j$. Regarding the FSO links, the receiver radius, beam waist and pointing error displacement standard deviation are assumed to be the same for all links and they will be denoted by a, ω_z and σ_s, respectively. In what follows, we set $\sigma_s/a = 3$. The values of ω_z/a will be varied in the simulations where large values of this ratio indicate less pointing errors. For simplicity of notation, we define $\mathbf{d}_n \triangleq (d_{0,n}, d_{n,N+1})$ where all distances are expressed in km.

Fig. 2 shows the performance with 2, 4 and 6 relays in the absence of pointing errors ($\omega_z/a \to \infty$). We assume that all RF links are subject to Rayleigh fading and we highlight the impact of changing the RF SNR Ω. For $N = 2$, the relays are placed at $\mathbf{d}_1 = (1.5, 2.5)$ and $\mathbf{d}_2 = (2.5, 1.5)$. For $N = 4$, two additional relays are added at $\mathbf{d}_3 = (1.25, 2.25)$ and $\mathbf{d}_4 = (2, 1)$. For $N = 6$, two extra relays are placed at $\mathbf{d}_5 = (1.5, 1.5)$ and $\mathbf{d}_6 = (2.25, 1.25)$. Results highlight the enhanced performance levels and diversity orders that are achieved by the proposed scheme for different numbers of relays. For $N = 2$ at an outage probability of 10^{-10}, the proposed cooperation strategy outperforms parallel-relaying [9, 10] by about 0.88 dB, 2.25 dB and 3.15 dB for $\Omega = 10$ dB, $\Omega = 20$ dB and $\Omega = 30$ dB, respectively. For $N = 4$ and $N = 6$, results show that the performance does not improve by increasing Ω beyond 20 dB and 10 dB, respectively. This is justified by the fact that at least $N - 1$ RF outage probability terms multiply each FSO outage probability term as highlighted in Section 3.3. In this context, even if the individual RF outage probabilities are not very small, their multiplication will result in a $(N - 1)$-fold decrease which is more significant for large values of N. In this scenario, small RF SNRs are sufficient for attaining the full potential of the inter-relay RF links.

As highlighted in Fig. 2, activating the RF links results in significant diversity gains even with 6-relay systems that initially profit from high diversity orders with parallel-relaying. As predicted by (32) and (33), the diversity order increases from 18.95 to 25.75 by fully exploiting the presence of the RF links according to the proposed cooperation strategy. Finally, results highlight the accuracy of the upper-bound in (25) in predicting the performance for average-to-large values of the SNR. In fact, the proposed bound is extremely close to the exact outage probability and, under the different scenarios, the approximate and exact curves are barely distinguishable for outage probabilities below 10^{-3}.

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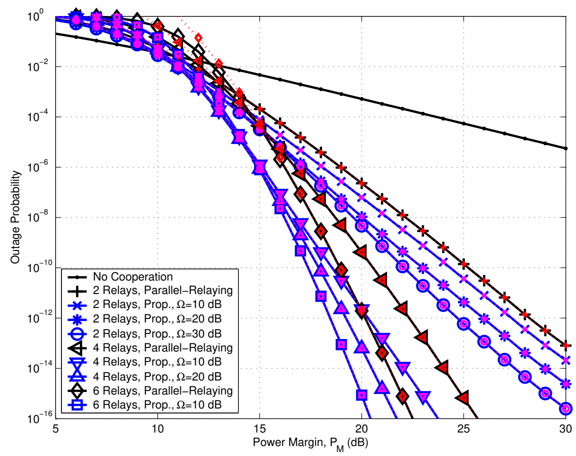

Fig. 2. Performance of the proposed scheme with 2, 4 and 6 relays. Solid lines correspond to the exact outage probability while the associated dotted lines correspond to the upper-bound. The same markers are used for both the exact outage probability and its corresponding upper-bound.

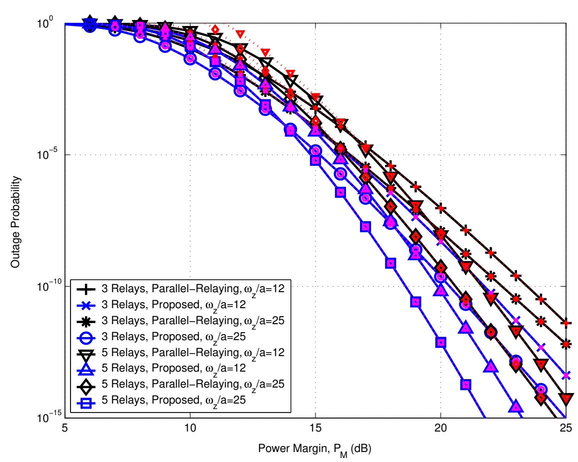

Fig. 3. Performance of the proposed scheme with 3 and 5 relays. Solid lines correspond to the exact outage probability while the associated dotted lines correspond to the upper-bound. The same markers are used for both the exact outage probability and its corresponding upper-bound.

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Fig. 3 shows the performance with 3 and 5 relays in the cases $\omega_z/a = 12$ and $\omega_z/a = 25$. The SNR along all RF links is fixed to $\Omega = 20$ dB. For $N = 3$, the relays are placed at $\mathbf{d}_1 = (1.5, 2.5)$, $\mathbf{d}_2 = (1, 2)$ and $\mathbf{d}_3 = (2.5, 1.5)$. In this case, all RF links are assumed to be subject to Rayleigh fading. For $N = 5$, two relays are added at $\mathbf{d}_4 = (1.45, 2.55)$ and $\mathbf{d}_5 = (2.55, 1.45)$. Given the proximity between R_1 and R_4 on one hand and between R_3 and R_5 on the other hand, the channel gains of the RF links R_1-R_4 and R_3-R_5 are taken to follow the Rician distribution with $K_{1,4} = K_{3,5} = 10$. Results show that significant performance gains can be obtained under different misalignment-fading conditions. For $N = 5$ at an outage probability of 10^{-10}, the performance gains with respect to the parallel-relaying scheme are in the order of 1.8 dB and 2 dB for $\omega_z/a = 12$ and $\omega_z/a = 25$, respectively. As in Fig. 2, results highlight the close match between the exact outage probability and the proposed upper-bound. For $N = 3$, the diversity order is enhanced from 8.2 in the absence of inter-relay cooperation to 9.78 and 10.88 in the presence of inter-relay cooperation for $\omega_z/a = 12$ and $\omega_z/a = 25$, respectively.

5. Conclusion

The presence of the backup RF links constitutes an additional degree of freedom that can be exploited for enhancing the performance of relay-assisted FSO systems. In this work, the RF links are used for establishing R-R communications and reducing the system outage probability in a simple and efficient manner. Both exact and asymptotic performance evaluations reflected the significant gains that can be achieved by the proposed cooperation strategy.