Impact of Inter-Relay Cooperation on the Performance of FSO Systems with any Number of Relays

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***Abstract*—In this paper, we study the impact of inter-relay cooperation on the performance of Decode-and-Forward (DF) cooperative Free Space Optical (FSO) communication systems with any number of relays. The idea of inter-relay cooperation (IRC) was introduced very recently where the relay-relay links are activated for further boosting the system performance. We evaluate the outage probability under forward and forward-backward IRC that constitute the two variants of this transmission strategy. We also derive the diversity orders that can be achieved over a composite channel model that takes both turbulence-induced fading and misalignment-induced fading into consideration. We present a comprehensive asymptotic analysis that is effective for tackling the usefulness of IRC with an arbitrary number of relays and for deriving the network conditions under which implementing IRC in any of its variants can be beneficial for enhancing the diversity order of the FSO system. The introduced framework answers the question on what is the optimal solution for a particular FSO network (among the parallel-relaying solution with no IRC, forward IRC or forward-backward IRC).**

***Index Terms*—Free-Space Optics, FSO, cooperation, relaying, outage, diversity, gamma-gamma, pointing errors.**

I. Introduction

Cooperative Free Space Optical (FSO) communications promptly developed into a well established field of research. A large number of contributions investigated the cooperative communication techniques as efficient distributed solutions for mitigating the turbulence-induced fading that severely degrades the performance of FSO links. Several variants of the cooperative solutions were examined comprising the association of parallel-relaying and serial-relaying techniques with Amplify-and-Forward (AF) and Decode-and-Forward (DF) transmission strategies. In this context, all-optical solutions or solutions that involve optical-to-electrical conversion were envisaged with either all-active or selective relaying schemes that can be implemented in the absence and presence of channel state information (CSI), respectively. Numerous performance measures were utilized for quantifying the gains with respect to non-cooperative communications including the bit error rate (BER), the outage probability, the ergodic capacity, and the diversity-multiplexing tradeoff. Furthermore, a wide variety of fading models were adopted including the exponential, the lognormal, and the gamma-gamma models. More recent contributions included the misalignment-induced fading or pointing errors that result from the building sway.

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All-active parallel-relaying has also been widely investigated [1]–[14]. In parallel-relaying, a signal is first transmitted from the source to the destination and relays; at a second time, the relays retransmit the signal to the destination. All-active relaying constitutes an appealing solution to such systems where all relays participate in the cooperation effort irrespective of the network conditions that results in a simple solution that does not require acquiring the CSI at any node. In [1], all-active parallel-relaying was analyzed through an outage probability analysis over lognormal channels where the AF and the DF strategies were compared. Ref. [2] evaluated the BER performance of AF cooperation with one relay. The BER performance of DF cooperation with one relay was analyzed in [3]–[5] in the context of intensity-modulation and direct-detection, subcarrier intensity modulation and differential modulation, respectively. The outage probability and diversity-multiplexing tradeoff were evaluated in [6] in the context of parallel-relaying with one relay. Power allocation for all-active parallel-relaying FSO systems with any number of relays was investigated in [7]. DF schemes based on convolutional codes were proposed in [8] while detect-and-forward schemes were analyzed over gamma-gamma channels in the presence of pointing errors in [9]. The impact of pointing errors on all-active parallel-relaying systems was further investigated in [10] and [11] where all-optical solutions and solutions that involve optical-to-electrical conversion were analyzed, respectively. The DF parallel-relaying with an arbitrary number of relays was also investigated in [12] where two variants of the DF strategy were compared over gamma-gamma fading channels; in the first one, all symbols received by a relay are retransmitted to the destination while in the second scheme a selective process is applied on the symbols to be retransmitted. The problem of optimal relay placement was tackled in [13] and [14] where unconstrained optimization and constrained optimization with link obstacles and infeasible regions were performed, respectively.

While the all-active parallel-relaying systems in [1]–[14] correspond to a two-phase solution based on sequential source-relay (and eventually source-destination) followed by relay-destination communications, the idea of inter-relay cooperation (IRC) was introduced and analyzed in [15] and [16]. For such systems, the relays cooperate with each other before the retransmission phase towards the destination that enhances the fidelity of signal reconstruction at the relays and eventually boosts the performance of the cooperative FSO network. In this context, the solution in [15], [16] corresponds to a three-phase source-relay, relay-relay, and relay-destination cooperation strategy (further details on inter-relay cooperation can be found in subsection II-A below). In [15], the conditional BER and the optimal power allocation strategy were derived for any number of relays. In [16], a two-relay system was analyzed in the absence and presence of CSI. Two variants of the inter-relay cooperation strategy were proposed and their outage probabilities and diversity orders were derived over gamma-gamma fading channels.

In this work, we extend the outage probability analysis presented in [16] with two relays to an arbitrary number of relays in the absence of CSI. We derive the exact outage probability expressions and the diversity orders that can be achieved by the two variants IRC1 and IRC2 where IRC is implemented in the forward and forward-backward directions, respectively. The second contribution of the paper resides in a comprehensive analysis on the utility of inter-relay cooperation. In this context, we highlight the conditions under which IRC1 or IRC2 can improve the diversity order of a particular FSO network. In particular, depending on the state of the source-relay and relay-destination links, we propose an efficient and simple framework that allows to determine whether (i) neither IRC1 nor IRC2, (ii) only IRC2 or (iii) both IRC1 and IRC2 can enhance the diversity order with respect to systems that do not implement IRC. In the last case, the comparison between IRC1 and IRC2 depends on the state of the relay-relay links as well. We propose a technique for analytically comparing IRC1 and IRC2 under this scenario if a certain number of network conditions is satisfied.

II. System Model

A. Cooperation Strategies

Consider a relay-assisted FSO communication system where N relays are assumed to be present in the vicinity of a source node S and a destination node D. The relay nodes correspond to independent communication entities that are initially deployed for ensuring wireless optical connectivity between different locations. In case these nodes have no information to communicate, they can serve as relays for assisting S in its communication with D. This constitutes a major advantage of cooperative systems where no additional infrastructure needs to be deployed. In what follows, the relays will be denoted by R_1, ..., R_N. For simplicity of notation, S and D will be denoted by R_0 and R_{N+1}, respectively.

We will analyze and compare the three following cooperation strategies: the No Inter-Relay-Cooperation scheme (NIRC), One-way IRC scheme (IRC1), and Two-way IRC scheme (IRC2). It is worth noting that all of these schemes can be implemented in the absence of CSI at the destination and the relays that renders them suitable for simple noncoherent communications based on Intensity-Modulation and Direct-Detection (IM/DD). The considered cooperation strategies are based on the DF relaying scheme. In this context, the signal is first decoded at each relay followed by a re-encoding/retransmission phase. The first step involves optical-to-electrical conversion while the second step involves electrical-to-optical conversion.

1) NIRC: NIRC corresponds to the conventional two-phase all-active parallel-relaying scheme often considered in the literature. For NIRC, S first transmits the information message to D and the relays and, at a second time, the relays retransmit this message to D.

2) IRC: For the IRC schemes, after the first communication phase, the relays inter-cooperate with each other to enhance the fidelity of the reconstructed symbols before the retransmission phase to D. This inter-relay cooperation can be realized either in unidirectional or in bidirectional manners resulting in two variants of this strategy; namely, IRC1 and IRC2. (i): For IRC1, each relay retransmits the message to the next relay (if any). In other words, the decision at R_n will be based on the signals received from S and R_{n-1} (if any). (ii): For IRC2, *forward-backward* inter-relay cooperation is envisaged where the decision at R_n will be based on the signals received from S, R_{n-1}, and R_{n+1} (if any).

The transmission procedure is as follows. For IRC1, the signal is first transmitted from S to all relays (and D) in one time slot. The relays then perform the following operation in a sequential manner for $n = 1, \dots, N$: if R_n successfully decodes at least one of the signals it received along the S-R_n link or the R_{n-1}-R_n link (if any), then R_n retransmits this message to R_{n+1} (if any); otherwise, R_n remains idle. The communications along the R_1-$R_2, \dots,$ R_{N-1}-R_N links occur sequentially over $N-1$ time slots. Finally, the relays that have successfully decoded the message retransmit this message to D in the last time slot. For IRC2, after triggering the communications over the R_n-R_{n+1} links, the relays perform the following operation in a sequential manner for $n = N, \dots, 1$: if R_n has successfully decoded the message in any of the previous slots, then R_n sends the information message to R_{n-1} (if any); otherwise, this backward communication does not take place. Potential communications along the links R_N-$R_{N-1}, \dots,$ R_2-R_1 necessitate $N-1$ additional slots before the final retransmission to D.

Evidently, the IRC schemes entail a higher system complexity as is the case of almost all advanced communication techniques. For example, the well explored NIRC scheme results in significant performance gains with respect to point-to-point communications; however, this improvement is associated with an increased complexity since the communications now involve $2N$ additional source-relay and relay-destination links. Similar to the improvement of NIRC with respect to non-cooperative transmissions, the considered IRC system further improves over the NIRC scheme by means of communicating over the relay-relay links. In cases where IRC is useful, implementing this system or not depends on the targeted levels of compromise between performance and complexity. It is worth noting that the additional complexity is limited to the signaling procedure to control the additional relay-relay communication phase as has been highlighted above without affecting the network infrastructure. In this context, no additional transceivers are added and no major hardware modifications are imposed on the existing transceivers except for an additional switching component that simply switches

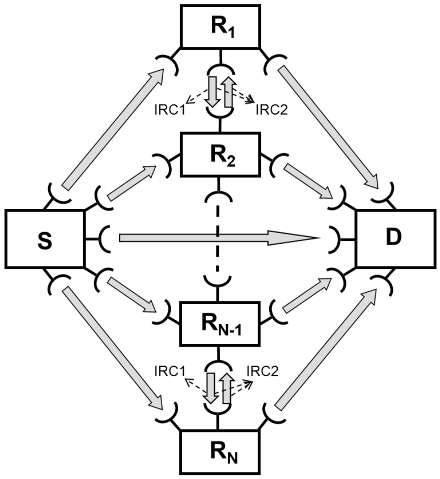

Fig. 1. FSO relay-assisted transmission with N inter-connected relays.

to the strongest transceiver as will be explained later. Moreover, following from the high directivity of the FSO links, the different transmissions do not interfere with each other thus bypassing all forms of involved joint encoding/decoding. In this context, triggering communications over the existing relay-relay links is not associated with any decoding complexity where the detection procedure at each transceiver remains the same compared to non-cooperative communications. The inter-relay communication phase simply incurs an additional decoding delay that can be straightforwardly compensated for at D. Finally, it is worth noting that the considered IRC schemes are noncoherent and, consequently, the extension of the existing NIRC scheme to the IRC schemes is not associated with any additional complexity for acquiring the CSI of the inter-relay links.

Fig. 1 illustrates a relay-assisted FSO network with IRC. Nodes S, D, R_1, ..., R_N correspond to buildings on which several transceivers are installed each of which ensures a directive FSO link with a neighboring building. The IRC schemes take advantage of the relay-relay links to boost the performance of the network. Implementing user cooperation with the existing infrastructure restrains the freedom of reallocating, redistributing or realigning any of the already existing transceivers and, in particular, the transceivers used for the inter-relay communications. For example, these $2(N-1)$ transceivers can not be used to create additional links between S and D via the relays since this will deprive the relay nodes from the possibility of communicating their information one with the other in the non-cooperative mode of the network. An information-carrying signal falls on each one of the different transceivers installed at a given relay. In this context, it is sufficient that at least one of these signals has a signal-to-noise ratio (SNR) that exceeds the decoding threshold to ensure the delivery of the information message to the relay. This significantly simplifies the implementation of the cooperative network where each relay simply switches to the *strongest* transceiver without further complications in the hardware as compared to non-cooperative systems. This also simplifies the shifting from the *cooperative mode* (where the relay is transmitting information of S) to the *non-cooperative mode* (where the relay is transmitting its own information). Finally, the number of signals that fall on each relay depends on the cooperation scheme and on the index of the relay. For IRC1, one signal is available at R_1 while two signals are available at each one of the relays R_2, ..., R_N. For IRC2, two signals are available at each one of the relays R_1 and R_N while three signals are available at each one of the remaining relays R_2, ..., R_{N-1}. Following from the high directivity of FSO links, the optical signal transmitted along the link R_i-R_j does not interfere with the signals transmitted along the other links. In particular, communicating over the relay-relay links does not incur any additional interference since the signal transmitted from a relay to the previous or next relay can not be overheard by other nodes in the network.

B. Channel Model

Denote by $I_{i,j}$ the irradiance along the link R_i-R_j. This irradiance can be written as the product of three terms: $I_{i,j} = I_{i,j}^{(l)} I_{i,j}^{(a)} I_{i,j}^{(p)}$ where, in this work, we adopt a channel model that takes into account the combined effects of path loss ($I_{i,j}^{(l)}$), atmospheric turbulence-induced scintillation ($I_{i,j}^{(a)}$) and misalignment-induced fading caused by pointing errors ($I_{i,j}^{(p)}$). Assuming a gamma-gamma turbulence model and a Gaussian spatial intensity profile falling on a circular aperture at the receiver, the probability density function (pdf) of $I_{i,j}$ was derived in [17] and expressed in terms of the Meijer G-function $G_{p,q}^{m,n}[.]$ as follows:

$$f_{I_{i,j}}(I) = \frac{\alpha_{i,j}\beta_{i,j}\xi_{i,j}^2}{A_{i,j} I_{i,j}^{(l)} \Gamma(\alpha_{i,j})\Gamma(\beta_{i,j})} \times G_{1,3}^{3,0}\left[\frac{\alpha_{i,j}\beta_{i,j}}{A_{i,j} I_{i,j}^{(l)}} I \middle| \begin{matrix} \xi_{i,j}^2 \\ \xi_{i,j}^2-1, \alpha_{i,j}-1, \beta_{i,j}-1 \end{matrix}\right] \quad (1)$$

where $\Gamma(.)$ is the Gamma function. In (1), $\alpha_{i,j}$ and $\beta_{i,j}$ stand for the parameters of the gamma-gamma distribution and can be written as follows:

$$\alpha_{i,j} = \left[\exp\left(0.49\sigma_{R,i,j}^2/(1+1.11\sigma_{R,i,j}^{12/5})^{7/6}\right) - 1\right]^{-1} \quad (2)$$

$$\beta_{i,j} = \left[\exp\left(0.51\sigma_{R,i,j}^2/(1+0.69\sigma_{R,i,j}^{12/5})^{5/6}\right) - 1\right]^{-1} \quad (3)$$

where $\sigma_{R,i,j}^2 = 1.23 C_n^2 k^{7/6} d_{i,j}^{11/6}$ is the Rytov variance where $d_{i,j}$ stands for the length of the link R_i-R_j, k is the wave number, and C_n^2 denotes the refractive index structure parameter.

In (1), the parameters $A_{i,j}$ and $\xi_{i,j}$ are related to the pointing errors. $A_{i,j}$ is given by $A_{i,j} = [\text{erf}(v_{i,j})]^2$ where erf(.) stands for the error function with $v_{i,j} = \sqrt{\pi/2}(a_{i,j}/\omega_{z,i,j})$ where $a_{i,j}$ is the radius of the receiver and $\omega_{z,i,j}$ is the beam waist along the link R_i-R_j. $\xi_{i,j} = \omega_{z_{eq},i,j}/2\sigma_{s,i,j}$ where $\sigma_{s,i,j}$ stands for the pointing error displacement standard deviation at the receiver and $\omega_{z_{eq},i,j}^2 = \omega_{z,i,j}^2 \sqrt{\pi}\text{erf}(v_{i,j})/[2v_{i,j}e^{-v_{i,j}^2}]$. Finally, the atmospheric loss is given by $I_{i,j}^{(l)} = e^{-\sigma d_{i,j}}$ where σ is the attenuation coefficient. $I_{i,j}^{(l)}$ is considered as a fixed scaling factor during a long period of time. Interested readers are referred to [17]–[19] for more details on the channel model.

C. Outage Probability along the Individual Links

The instantaneous electrical SNR along the link R$_i$-R$_j$ with IM/DD is given by [20]:

$$\gamma_{i,j} = \frac{\eta^2 I_{i,j}^2}{N_{\text{link}}^2 N_0} \tag{4}$$

where η is the optical-to-electrical conversion ratio and N_0 is the variance of the additive white Gaussian noise (AWGN).

In (4), N_{link} stands for the total number of links in the FSO network. The normalization by N_{link} ensures that the cooperative system transmits the same power as point-to-point non-cooperative systems. Given that the considered cooperation schemes can be implemented in the absence of CSI, then this transmit power will be evenly distributed among all available links; in other words, each FSO link will be allocated a fraction N_{link} of the total available transmit power. For NIRC, $N_{\text{link}} = 2N + 1$ counting for the N S-R links, N R-D links, and the direct S-D link. For IRC1, $N_{\text{link}} = 3N$ taking into consideration the additional $N - 1$ inter-relay links. Finally, for IRC2, $N_{\text{link}} = 4N - 1$ since the $N - 1$ inter-relay links can be activated in both directions. It is worth noting that the potential performance gains associated with IRC result from the additional number of links used for communication. This is analogous to any other spatial diversity technique where the performance gains follow from diversifying the paths along which the signal propagates from S to D. In this context, the problem of designing cooperative networks can be formulated as follows: for a given network with a fixed number of relays that fixes the system's hardware complexity, what is the best cooperation strategy that can be implemented for achieving the highest performance levels without transmitting more power?

Given that the mean of the random variable $I_{i,j}$, having a pdf as given in (1), is equal to $A_{i,j} I_{i,j}^{(l)} \xi_{i,j}^2 / (\xi_{i,j}^2 + 1)$ [17], then the electrical SNR for the direct non-cooperative transmission along S-D can be written as [19], [20]:

$$\overline{\gamma}_{0,N+1} = \frac{\eta^2 \text{E}^2[I_{0,N+1}]}{N_0} = \frac{1}{N_0} \left(\frac{\eta A_{0,N+1} I_{0,N+1}^{(l)} \xi_{0,N+1}^2}{\xi_{0,N+1}^2 + 1} \right)^2 \tag{5}$$

where E[.] stands for the averaging operator.

Consequently, (4) can be written as: $\gamma_{i,j} = \overline{\gamma}_{0,N+1} \left(\frac{\xi_{0,N+1}^2 + 1}{A_{0,N+1} I_{0,N+1}^{(l)} \xi_{0,N+1}^2} \right)^2 \left(\frac{I_{i,j}}{N_{\text{link}}} \right)^2$.

The link R$_i$-R$_j$ is in outage if the SNR $\gamma_{i,j}$ falls below a specified decoding threshold denoted by γ_{th} above which the signal can be decoded with an arbitrarily small probability of error. The outage probability $p_{i,j} \triangleq \Pr(\gamma_{i,j} < \gamma_{th})$ along this link can be written as:

$$p_{i,j} = \Pr \left(I_{i,j} < \frac{N_{\text{link}}}{\mathcal{P}_M} \frac{A_{0,N+1} I_{0,N+1}^{(l)} \xi_{0,N+1}^2}{\xi_{0,N+1}^2 + 1} \right) \tag{6}$$

where $\mathcal{P}_M = \sqrt{\frac{\overline{\gamma}_{0,N+1}}{\gamma_{th}}}$ denotes the optical power margin.

The cumulative distribution function (cdf) associated with the pdf in (1) can be expressed in terms of the Meijer G-function as follows [17]:

$$F_{I_{i,j}}(I) = \frac{\xi_{i,j}^2}{\Gamma(\alpha_{i,j})\Gamma(\beta_{i,j})} G_{2,4}^{3,1} \left[\frac{\alpha_{i,j}\beta_{i,j}}{A_{i,j} I_{i,j}^{(l)}} I \middle| \begin{matrix} 1, \xi_{i,j}^2 + 1 \\ \xi_{i,j}^2, \alpha_{i,j}, \beta_{i,j}, 0 \end{matrix} \right] \tag{7}$$

Consequently, (6) can be written as:

$$p_{i,j} = F_{I_{i,j}} \left(\frac{N_{\text{link}}}{\mathcal{P}_M} \frac{A_{0,N+1} I_{0,N+1}^{(l)} \xi_{0,N+1}^2}{\xi_{0,N+1}^2 + 1} \right) \tag{8}$$

D. Diversity Order along the Individual Links

Equation (8) does not offer intuitive insights on the behavior of $p_{i,j}$. Consequently, we will further proceed with an asymptotic analysis. For large SNRs, the outage probability is dominated by the behavior of the pdf near the origin where (1) can be approximated by $f_{I_{i,j}}(I) \approx a_{i,j} I^{\zeta_{i,j} - 1}$ where $\zeta_{i,j} = \min\{\beta_{i,j}, \xi_{i,j}^2\}$ and:

$$a_{i,j} = \frac{\xi_{i,j}^2 (\alpha_{i,j}\beta_{i,j})^{\zeta_{i,j}} \Gamma(\alpha_{i,j} - \zeta_{i,j})}{(A_{i,j} I_{i,j}^{(l)})^{\zeta_{i,j}} \Gamma(\alpha_{i,j}) \Gamma(\beta_{i,j})} b_{i,j} \tag{9}$$

where $b_{i,j} = 1/(\xi_{i,j}^2 - \beta_{i,j})$ if $\xi_{i,j}^2 > \beta_{i,j}$ and $b_{i,j} = \Gamma(\beta_{i,j} - \xi_{i,j}^2)$ if $\xi_{i,j}^2 < \beta_{i,j}$ [21].

Based on the above approximation, the outage probability in (8) can be approximated by the following expression for large values of $\mathcal{P}_M$:

$$p_{i,j} \approx \frac{a_{i,j}}{\zeta_{i,j}} \left(\frac{\xi_{0,N+1}^2 + 1}{A_{0,N+1} I_{0,N+1}^{(l)} \xi_{0,N+1}^2} \frac{\mathcal{P}_M}{N_{\text{link}}} \right)^{-\zeta_{i,j}} \tag{10}$$

that scales asymptotically as $\mathcal{P}_M^{-\zeta_{i,j}}$ showing that the diversity order along the link R$_i$-R$_j$ is equal to $\zeta_{i,j} = \min\{\beta_{i,j}, \xi_{i,j}^2\}$ in coherence with the asymptotic analysis presented in [22] for gamma-gamma channels with pointing errors. Finally, we assume that the FSO channels are reciprocal resulting in $I_{i,j} = I_{j,i}$ and $p_{i,j} = p_{j,i}$.

III. Outage Probability

From (7) and (8), the outage probability of the overall FSO system depends on the power margin $\mathcal{P}_M$, on the network setup (through the channel parameters $\alpha_{i,j}$, $\beta_{i,j}$, $\xi_{i,j}$, $A_{i,j}$ and $I_{i,j}^{(l)}$), and on the number of relays N. In order to offer more insights on the performance of IRC1 and IRC2, we first consider the special cases of $N = 2$ and $N = 3$.

A. Special cases

1) $N = 2$: Considering all possible conditions of the S-R links, the outage probability of the cooperative FSO network can be written under the following general form:

$$P_{\text{out}} = p_{0,3} \left[q_{0,1} q_{0,2} Q_0 + p_{0,1} p_{0,2} Q_1 + q_{0,1} p_{0,2} Q_2 + p_{0,1} q_{0,2} Q_3 \right] \tag{11}$$

where $q_{i,j} \triangleq 1 - p_{i,j}$ is the probability that the link R$_i$-R$_j$ is not in outage. In (11), the multiplication by $p_{0,3}$ results from the fact that the system will not be in outage if the direct link S-D (denoted equivalently by R$_0$-R$_3$) is not in outage.

The probabilities $Q_0, \ldots, Q_3$ in (11) depend on the implemented cooperation scheme:

– Q_0: In this case, the information symbol is available at both relays since the links S-R$_1$ and S-R$_2$ are not in outage. Consequently, the system will be in outage if the links R$_1$-D and R$_2$-D are in outage. Consequently, $Q_0 = p_{1,3}p_{2,3}$ whether with IRC1 or with IRC2.

– Q_1: Both relays are in outage and $Q_1 = 1$ for IRC1 and IRC2.

– Q_2: In this case, the information symbol is available at R$_1$ that is not in outage and, hence, for the system to be in outage, the link R$_1$-D must be in outage. For the IRC schemes, even though R$_2$ did not acquire the message from S, yet it can still acquire it from R$_1$. In other words, for both IRC1 and IRC2, since the message is available at R$_1$, then it can be forwarded along the additional path R$_1$-R$_2$-D where this *forward* cooperation is an additional degree of freedom that is exploited by the IRC protocols. In this case, $Q_2 = p_{1,3}p_{1\to2}$ where $p_{1\to2} \triangleq p_{1,2} + q_{1,2}p_{2,3}$. In fact, if R$_1$-R$_2$ is in outage (with probability $p_{1,2}$), then the information message can not reach R$_2$ (since in this case S-R$_2$ is in outage as well) and, consequently, R$_2$ can not retransmit to D. Otherwise, with probability $q_{1,2}$, R$_2$ is acquiring the message from R$_1$ and, for the system to be in outage, R$_2$-D must be in outage (with probability $p_{2,3}$).

– Q_3: In this case, only R$_2$ is acquiring the message from S and, hence, for the system to be in outage, the link R$_2$-D must be in outage. (i): For IRC1, R$_1$ can base its decision only on the signal received from S. Given that the link S-R$_1$ is in outage in this case, then R$_1$ can not participate in the cooperation effort resulting in $Q_3 = p_{2,3}$. (ii): For IRC2, since the message is available at R$_2$, then it can be forwarded along the additional path R$_2$-R$_1$-D where this *backward* cooperation is an additional degree of freedom that is exploited exclusively by IRC2. Consequently, $Q_3 = p_{2,3}p_{2\to1} = p_{2,3}(p_{2,1} + q_{2,1}p_{1,3})$.

2) $N = 3$: Similar to the case $N = 2$, the system outage probability can be written as:

$$P_{\text{out}} = p_{0,4}\left[q_{0,1}q_{0,2}q_{0,3}Q_0 + p_{0,1}q_{0,2}q_{0,3}Q_1 + q_{0,1}p_{0,2}q_{0,3}Q_2 + q_{0,1}q_{0,2}p_{0,3}Q_3 + p_{0,1}p_{0,2}q_{0,3}Q_4 + p_{0,1}q_{0,2}p_{0,3}Q_5 + q_{0,1}p_{0,2}p_{0,3}Q_6 + p_{0,1}p_{0,2}p_{0,3}Q_7\right] \quad (12)$$

The probabilities $Q_0, \ldots, Q_7$ can be calculated as follows.

– Q_0: Similar to the case $N = 2$, $Q_0 = p_{1,4}p_{2,4}p_{3,4}$ whether with IRC1 or with IRC2 since all R-D links must be in outage for the system to be in outage.

– Q_1: In this case, the information symbol is available at R$_2$ and R$_3$ that are not in outage and, hence, for the system to be in outage, the links R$_2$-D and R$_3$-D must be in outage.

- For IRC1, $Q_1 = p_{2,4}p_{3,4}$ since R$_1$ can not participate in the cooperation effort.
- For IRC2, since the message is available at R$_2$, then it can be forwarded along the additional path R$_2$-R$_1$-D resulting in $Q_1 = p_{2,4}p_{3,4}p_{2\to1} = p_{2,4}p_{3,4}(p_{2,1} + q_{2,1}p_{1,4})$.

– Q_2: In this case, S-R$_1$ and S-R$_3$ are not in outage and, consequently, R$_1$-D and R$_3$-D must fail so that the entire network will suffer from outage.

- For IRC1, while the link S-R$_2$ is in outage, yet R$_2$ can still acquire the information message from R$_1$. Therefore, $Q_2 = p_{1,4}p_{3,4}p_{1\to2} = p_{1,4}p_{3,4}\left[p_{1,2} + q_{1,2}p_{2,4}\right]$.
- For IRC2, R$_2$ can acquire the information message from either R$_1$ or R$_3$ (or both). In this case, Q_2 can be written under the form $Q_2 = p_{1,4}p_{3,4}p_{1\to2\leftarrow3}$ where $p_{1\to2\leftarrow3} \triangleq p_{2,4} + q_{2,4}(p_{1,2}p_{3,2})$. In fact, if the link R$_2$-D is in outage (with probability $p_{2,4}$), then even if R$_2$ acquires and retransmits the information message, this message will not reach D. Otherwise, a message retransmitted from R$_2$ will reach D via R$_2$-D (with probability $q_{2,4}$). In this case, R$_2$ will not acquire the message if the two inter-relay links R$_1$-R$_2$ and R$_3$-R$_2$ fail simultaneously (with probability $p_{1,2}p_{3,2}$). Note that, in this case, Q_2 can not be written as $Q_2 = p_{1,4}p_{3,4}p_{1\to2}p_{3\to2}$ since the paths R$_1$-R$_2$-D and R$_3$-R$_2$-D are not independent.

– Q_3: In this case, S-R$_1$ and S-R$_2$ are not in outage. In a way similar to the previous case, $Q_3 = p_{1,4}p_{2,4}p_{2\to3} = p_{1,4}p_{2,4}\left[p_{2,3} + q_{2,3}p_{3,4}\right]$ for both IRC1 and IRC2 where, in this case, R$_3$ can acquire the message from R$_2$.

– Q_4: In this case, S-R$_1$ and S-R$_2$ are in outage.

- For IRC1, R$_1$ and R$_2$ can not participate in the cooperation effort since they can not receive the information message from the subsequent relay R$_3$ resulting in $Q_4 = p_{3,4}$.
- For IRC2, R$_3$ is the only relay that is acquiring the message from S. This message can be subsequently forwarded to D either directly along R$_3$-D or indirectly via one of the relays R$_2$ or R_1. In this case, Q_4 can be written as $Q_4 = p_{3,4}p_{3\to2\to1}$ where $p_{3\to2\to1} = p_{3,2} + q_{3,2}p_{2,4}p_{2\to1}$. In fact, if the link R$_3$-R$_2$ is in outage, then the information message can not reach R$_2$ which in turn can not forward this message to R$_1$ resulting in an outage of the system. Otherwise, the message is available at R$_2$ that can forward this message either along the direct link R$_2$-D or along the indirect link R$_2$-R$_1$-D. Consequently, $Q_4 = p_{3,4}\left[p_{3,2} + q_{3,2}p_{2,4}\left(p_{2,1} + q_{2,1}p_{1,4}\right)\right]$.

– Q_5: In this case, R$_2$ is the only relay that is acquiring the information message from S.

- For IRC1, the acquired message can be forwarded from R$_2$ to D via the paths R$_2$-D or R$_2$-R$_3$-D resulting in $Q_5 = p_{2,4}p_{2\to3} = p_{2,4}\left[p_{2,3} + q_{2,3}p_{3,4}\right]$.
- For IRC2, the additional path R$_2$-R$_1$-D is available resulting in $Q_5 = p_{2,4}p_{2\to3}p_{2\to1} = p_{2,4}\left[p_{2,3} + q_{2,3}p_{3,4}\right]\left[p_{2,1} + q_{2,1}p_{1,4}\right]$.

– Q_6: In this case, R$_1$ is the only relay that is acquiring the message that can be forwarded to D either directly along R$_1$-D or indirectly via one of the relays R$_2$ or R_3. In this case, $Q_6 = p_{1,4}p_{1\to2\to3} = p_{1,4}\left[p_{1,2} + q_{1,2}p_{2,4}\left(p_{2,3} + q_{2,3}p_{3,4}\right)\right]$ whether with IRC1 or with IRC2.

– Q_7: Finally, $Q_7 = 1$ for both IRC1 and IRC2 since all the relays are in outage in this case.

For NIRC, all the probabilities that arise from inter-relay cooperation must be set to 1 where the involved probabilities are $\{p_{n\to(n+1)}, p_{(n+1)\to n}\}_{n=1}^{2}$, $p_{1\to2\to3}$, $p_{3\to2\to1}$, and $p_{1\to2\leftarrow3}$.

B. IRC with N Relays

After introducing the different probability definitions and highlighting on the cases that might arise in IRC systems with $N = 3$, we next tackle the general case of IRC systems with

$N \geq 2$. The outage probability of the overall FSO system can be written as:

$$P_{\text{out}} = \sum_{\mathcal{S} \subset \{1,\dots,N\}} \Pr\left(\{R_j\}_{j\in\mathcal{S}} \text{ not in outage}\right) \times \Pr\left(\text{system in outage} \mid \{R_j\}_{j\in\mathcal{S}} \text{ not in outage}\right) \quad (13)$$

where the first probability can be derived as $\left[\prod_{j\in\mathcal{S}} q_{0,j}\right]\left[\prod_{j'\in\overline{\mathcal{S}}} p_{0,j'}\right]$ since when the relays in $\mathcal{S}$ are not in outage, then the relays in $\overline{\mathcal{S}}$ will be in outage. The second probability can be written as $p_{0,N+1}\left[\prod_{j\in\mathcal{S}} p_{j,N+1}\right] P_{\mathcal{S}}^{(\text{IRC})}$ because, for the system to be in outage, (i): the direct link must be in outage with probability $p_{0,N+1}$, (ii): the relays in $\mathcal{S}$ that succeeded in decoding the message from S must fail in delivering this message to D where the associated probability is $\prod_{j\in\mathcal{S}} p_{j,N+1}$, and (iii): IRC must fail in preventing outage where the corresponding probability is denoted by $P_{\mathcal{S}}^{(\text{IRC})}$. Indexing all possible subsets $\mathcal{S}$ of $\{1,\dots,N\}$ as $\mathcal{I}_{n,1},\dots,\mathcal{I}_{n,\binom{N}{n}}$, where $n \in \{0,\dots,N\}$ denotes the cardinality, results in the following expression of the outage probability:

$$P_{\text{out}} = p_{0,N+1} \sum_{n=0}^{N} \sum_{i=1}^{\binom{N}{n}} \left[\prod_{j\in\mathcal{I}_{n,i}} q_{0,j}\right]\left[\prod_{j'\in\overline{\mathcal{I}}_{n,i}} p_{0,j'}\right] \times \left[\prod_{j\in\mathcal{I}_{n,i}} p_{j,N+1}\right] P_{\mathcal{I}_{n,i}}^{(\text{IRC})} \quad (14)$$

where n corresponds to the number of relays that are not in outage. $\mathcal{I}_{n,1},\dots,\mathcal{I}_{n,\binom{N}{n}}$ are all possible subsets of $\{1,\dots,N\}$ having n elements each. Note that the probability $P_{\mathcal{I}_{n,i}}^{(\text{IRC})}$ results from implementing IRC where this probability must be set to 1 in the case of NIRC.

For example, for $N = 3$, the probabilities in (12) and (14) are related as follows: $Q_0 = p_{1,4}p_{2,4}p_{3,4}P_{\{1,2,3\}}^{(\text{IRC})}$, $Q_1 = p_{2,4}p_{3,4}P_{\{2,3\}}^{(\text{IRC})}$, $Q_2 = p_{1,4}p_{3,4}P_{\{1,3\}}^{(\text{IRC})}$, $Q_3 = p_{1,4}p_{2,4}P_{\{1,2\}}^{(\text{IRC})}$, $Q_4 = p_{3,4}P_{\{3\}}^{(\text{IRC})}$, $Q_5 = p_{2,4}P_{\{2\}}^{(\text{IRC})}$, $Q_6 = p_{1,4}P_{\{1\}}^{(\text{IRC})}$, and $Q_7 = P_{\phi}^{(\text{IRC})}$ where ϕ stands for the empty set.

Consider first the set $\mathcal{I}_{n,i}$ containing the indices of the relays that are not in outage (in terms of the signals received from S) and assume that the elements of this set are arranged in an increasing order. The following partitioning will be applied on $\mathcal{I}_{n,i}$ that will be written as:

$$\mathcal{I}_{n,i} = \mathcal{N}_{n,i}^{(1)} \cup \mathcal{N}_{n,i}^{(2)} \cup \cdots \cup \mathcal{N}_{n,i}^{(m_{n,i})} \quad (15)$$

where the elements of $\mathcal{I}_{n,i}$ are grouped into subsets where each subset corresponds to a cluster of consecutive integers. In a more formal way, $\mathcal{N}_{n,i}^{(j)}$ corresponds to the j-th subset of consecutive integers of $\mathcal{I}_{n,i}$.

In the same way, the set $\overline{\mathcal{I}}_{n,i}$ that contains the indices of the relays that are in outage (in an increasing order) can be partitioned as follows:

$$\overline{\mathcal{I}}_{n,i} = \mathcal{O}_{n,i}^{(1)} \cup \mathcal{O}_{n,i}^{(2)} \cup \cdots \cup \mathcal{O}_{n,i}^{(m'_{n,i})} \quad (16)$$

where $\mathcal{O}_{n,i}^{(j)}$ corresponds to the j-th subset of consecutive integers of $\overline{\mathcal{I}}_{n,i}$ and where the two following additional rules need to be applied:

$$\mathcal{O}_{n,i}^{(1)} = \phi \text{ if } \min\{\mathcal{O}_{n,i}^{(1)}\} \neq 1 \quad (17)$$

$$\mathcal{O}_{n,i}^{(m'_{n,i})} = \phi \text{ if } \max\{\mathcal{O}_{n,i}^{(m'_{n,i})}\} \neq N \quad (18)$$

For example, assume that $N = 5$ and $\mathcal{I}_{n,i} = \{3,5\}$, then this set will be written as: $\mathcal{I}_{n,i} = \{3\}\cup\{5\}$. In this case, $\overline{\mathcal{I}}_{n,i} = \{1,2,4\}$ that can be partitioned as: $\overline{\mathcal{I}}_{n,i} = \{1,2\}\cup\{4\}\cup\phi$. If $N = 8$ with $\mathcal{I}_{n,i} = \{1,4,7,8\}$ and $\overline{\mathcal{I}}_{n,i} = \{2,3,5,6\}$, then $\mathcal{I}_{n,i} = \{1\}\cup\{4\}\cup\{7,8\}$ and $\overline{\mathcal{I}}_{n,i} = \phi\cup\{2,3\}\cup\{5,6\}\cup\phi$.

Based on the above, the relays can be partitioned into alternating subsets of *in-outage* and *not-in-outage* groups:

$$\{1,\dots,N\} = \mathcal{O}_{n,i}^{(1)} \cup \mathcal{N}_{n,i}^{(1)} \cup \cdots \mathcal{O}_{n,i}^{(m_{n,i})} \cup \mathcal{N}_{n,i}^{(m_{n,i})} \cup \mathcal{O}_{n,i}^{(m_{n,i}+1)} \quad (19)$$

where it can be easily proven that $m'_{n,i} = m_{n,i} + 1$.

1) IRC1: In this case, given that the inter-relay cooperation is implemented only in the *forward* direction, then inter-relay cooperation will not benefit the first group of relays that were already in outage before the inter-relay cooperation phase (whose indices are given in $\mathcal{O}_{n,i}^{(1)}$) where these relays will remain in outage after inter-cooperation between the relays. On the other hand, relays whose indices fall in $\mathcal{O}_{n,i}^{(k+1)}$ for $k > 0$ can benefit from inter-relay cooperation since these relays can receive the information message from the previous cluster of relays that are not in outage; i.e., from the relays whose indices fall in $\mathcal{N}_{n,i}^{(k)}$.

Denote by $\mathcal{N}_{n,i,l}^{(k)}$ the l-th element of $\mathcal{N}_{n,i}^{(k)}$ and by $|\mathcal{C}|$ the cardinality of the set $\mathcal{C}$. Given that inter-relay cooperation is envisaged in the *forward* direction and that the elements of $\mathcal{N}_{n,i}^{(k)}$ are arranged in increasing order, then the states of the links $\text{R}_{\mathcal{N}_{n,i,1}^{(k)}}$-$\text{R}_{\mathcal{N}_{n,i,2}^{(k)}}$, …, $\text{R}_{\mathcal{N}_{n,i,|\mathcal{N}_{n,i}^{(k)}|-1}^{(k)}}$-$\text{R}_{\mathcal{N}_{n,i,|\mathcal{N}_{n,i}^{(k)}|}^{(k)}}$ does not affect the outage probability since the information symbol is already available at $\mathcal{N}_{n,i,|\mathcal{N}_{n,i}^{(k)}|}^{(k)} = \max\{\mathcal{N}_{n,i}^{(k)}\}$ (from S).

Based on the above, and considering the clusters $\mathcal{N}_{n,i}^{(k)}$ and $\mathcal{O}_{n,i}^{(k+1)}$, the information message can propagate sequentially from $\text{R}_{\max\{\mathcal{N}_{n,i}^{(k)}\}}$ to $\text{R}_{\mathcal{O}_{n,i,1}^{(k+1)}}$ to $\text{R}_{\mathcal{O}_{n,i,2}^{(k+1)}}$ … to $\text{R}_{\max\{\mathcal{O}_{n,i}^{(k+1)}\}}$. In this case, the state of the link $\text{R}_{\max\{\mathcal{O}_{n,i}^{(k+1)}\}}$-$\text{R}_{\mathcal{N}_{n,i,1}^{(k+1)}}$ (if any) does not affect the outage probability since the information message is already acquired at $\text{R}_{\mathcal{N}_{n,i,1}^{(k+1)}}$ since the corresponding link with S is not in outage. Therefore, based on the above analysis, the probability $P_{\mathcal{I}_{n,i}}^{(\text{IRC})}$ can be written as:

$$P_{\mathcal{I}_{n,i}}^{(\text{IRC1})} = \prod_{k=1}^{m_{n,i}} p_{\max\{\mathcal{N}_{n,i}^{(k)}\}\to\min\{\mathcal{O}_{n,i}^{(k+1)}\}\to\cdots\to\max\{\mathcal{O}_{n,i}^{(k+1)}\}} \quad (20)$$

where the probability $p_{i\to i+1\to\cdots\to i+f}$ stands for the probability that the information message can not be delivered from any of the relays R_{i+1}, …, R_{i+f} to D where each one of these relays, in its turn, can acquire the message from the previous relay (since the corresponding link with S is in outage). Probabilities of this form can be calculated in a recursive manner as follows:

$$p_{i\to i+1\to\cdots\to i+f} = p_{i,i+1} + q_{i,i+1}p_{i+1,N+1}p_{i+1\to i+2\to\cdots\to i+f} \tag{21}$$

with $p_{i\to i} \triangleq 1$.

In fact, if the link R$_i$-R$_{i+1}$ is in outage (with probability $p_{i,i+1}$), then the propagation of the signal along R$_i$-R$_{i+1}$-$\cdots$-R$_{i+f}$ is stopped. In this case, none of the relays R$_{i+1}$, ..., R$_{i+f}$ can acquire the information message and, hence, none of these relays can forward this message to D. If the link R$_i$-R$_{i+1}$ is not in outage (with probability $q_{i,i+1}$), then the information message will be available at R$_{i+1}$. In this case, the message can not reach D only if the link R$_{i+1}$-D is in outage (with probability $p_{i+1,N+1}$) and none of the subsequent relays R$_{i+2}$, ..., R$_{i+f}$ can deliver the message to D (with probability $p_{i+1\to i+2\to\cdots\to i+f}$).

Regarding the previous example, for $N = 5$, $P^{(\text{IRC1})}_{\{3,5\}} = p_{3\to4} = p_{3,4} + q_{3,4}p_{4,6}$. For $N = 8$, $P^{(\text{IRC1})}_{\{1,4,7,8\}} = p_{1\to2\to3}p_{4\to5\to6} = [p_{1,2} + q_{1,2}p_{2,9}(p_{2,3} + q_{2,3}p_{3,9})][p_{4,5} + q_{4,5}p_{5,9}(p_{5,6} + q_{5,6}p_{6,9})]$.

Equation (20) can be written in an equivalent form as follows:

$$P^{(\text{IRC1})}_{\mathcal{I}_{n,i}} = \prod_{l=1}^{|\mathcal{I}_{n,i}|} p_{\mathcal{I}_{n,i,l}\to(\mathcal{I}_{n,i,l}+1)\to\cdots\to(\mathcal{I}_{n,i,l+1}-1)} \tag{22}$$

where $\mathcal{I}_{n,i,|\mathcal{I}_{n,i}|+1} \triangleq N + 1$.

2) IRC2: Regarding the relays whose indices fall in $\mathcal{O}^{(1)}_{n,i}$, these relays can profit from the presence of the information message at $\mathcal{N}^{(1)}_{n,i,1} = \min\{\mathcal{N}^{(1)}_{n,i}\}$ where this message can propagate in the *backward* direction: R$_{\min\{\mathcal{N}^{(1)}_{n,i}\}}$-R$_{\max\{\mathcal{O}^{(1)}_{n,i}\}}$-$\cdots$-R$_{\min\{\mathcal{O}^{(1)}_{n,i}\}}$. In this case, the relays in $\mathcal{O}^{(1)}_{n,i}$ will fail to deliver the message to D with probability $p_{\min\{\mathcal{N}^{(1)}_{n,i}\}\to\max\{\mathcal{O}^{(1)}_{n,i}\}\to\cdots\to\min\{\mathcal{O}^{(1)}_{n,i}\}}$ where:

$$p_{i\to i-1\to\cdots\to i-f} = p_{i,i-1} + q_{i,i-1}p_{i-1,N+1}p_{i-1\to i-2\to\cdots\to i-f} \tag{23}$$

in a way that is completely analogous to (21).

In a similar way, the group of relays whose indices fall in $\mathcal{O}^{(m_{n,i}+1)}_{n,i}$ can profit from the presence of the message at $\max\{\mathcal{N}^{(m_{n,i})}_{n,i}\}$ where this message can propagate in the *forward* direction. In this case, the inter-relay outage probability is $p_{\max\{\mathcal{N}^{(m_{n,i})}_{n,i}\}\to\min\{\mathcal{O}^{(m_{n,i}+1)}_{n,i}\}\to\cdots\to\max\{\mathcal{O}^{(m_{n,i}+1)}_{n,i}\}}$.

Consider now the relays whose indices fall in the set $\mathcal{O}^{(k)}_{n,i}$ for $k \in \{2, \ldots, m_{n,i}\}$. For IRC2, even though these relays were not capable of acquiring the information message from S, yet they can still acquire this message from the last relay in the previous group (i.e. R$_{\max\{\mathcal{N}^{(k-1)}_{n,i}\}}$) or from the first relay in the next group (i.e. R$_{\min\{\mathcal{N}^{(k)}_{n,i}\}}$). The probability that neither one of the relays in $\mathcal{O}^{(k)}_{n,i}$ is capable of delivering the message to D can be written as: $p_{\max\{\mathcal{N}^{(k-1)}_{n,i}\}\to\left(\min\{\mathcal{O}^{(k)}_{n,i}\}\rightleftarrows\cdots\rightleftarrows\max\{\mathcal{O}^{(k)}_{n,i}\}\right)\leftarrow\min\{\mathcal{N}^{(k)}_{n,i}\}}$ where probabilities of this form can be evaluated using the following recursive relation:

$$p_{i\to(i+1\rightleftarrows\cdots\rightleftarrows i+f-1)\leftarrow i+f} = \begin{cases} p_{i,i+1}p_{i+f\to i+f-1\to\cdots\to i+1} + \\ q_{i,i+1}p_{i+1,N+1}p_{i+1\to(i+2\rightleftarrows\cdots\rightleftarrows i+f-1)\leftarrow i+f}, & f > 1; \\ 1, & f = 1. \end{cases} \tag{24}$$

The interpretation of (24) is as follows. For $f > 1$, if the link R$_i$-R$_{i+1}$ is in outage, then the group of relays $\{i+1, \ldots, i+f-1\}$ can acquire the message exclusively from R$_{i+f}$ via backward cooperation. In this case, neither relay of this group will be able to deliver the message to D with probability $p_{i+f\to i+f-1\to\cdots\to i+1}$. On the other hand, with probability $q_{i,i+1}$, the link R$_i$-R$_{i+1}$ is not in outage and the message is now available at R$_{i+1}$. For the system to suffer from outage in this case, the link R$_{i+1}$-D must be in outage and the group of relays in $\{i+2, \ldots, i+f-1\}$ must fail in delivering the message to D where this group in its turn can acquire the message from either the previous relay R$_{i+1}$ or the next relay R$_{i+f}$. For $f = 1$, the information message is available at R$_i$ and R$_{i+f}$ =R$_{i+1}$ where no relays are present in between these relays resulting in a probability in (24) that is equal to 1. Note that for $f = 2$, $p_{i\to i+1\leftarrow i+2} = p_{i,i+1}p_{i+2\to i+1} + q_{i,i+1}p_{i+1,N+1}$ that is equal to the probability $p_{i+1,N+1} + q_{i+1,N+1}p_{i,i+1}p_{i+2,i+1}$ used in the previous subsection. In fact, if the link R$_{i+1}$-D is in outage then R$_{i+1}$ will fail in delivering the message to D irrespective of the states of the links R$_i$-R$_{i+1}$ and R$_{i+2}$-R$_{i+1}$. Otherwise, these two inter-relay links must both be in outage.

Based on the above analysis, $P^{(\text{IRC})}_{\mathcal{I}_{n,i}}$ is given by the following expression for IRC2:

$$P^{(\text{IRC2})}_{\mathcal{I}_{n,i}} = p_{\min\{\mathcal{N}^{(1)}_{n,i}\}\to\max\{\mathcal{O}^{(1)}_{n,i}\}\to\cdots\to\min\{\mathcal{O}^{(1)}_{n,i}\}}\times \prod_{k=2}^{m_{n,i}} p_{\max\{\mathcal{N}^{(k-1)}_{n,i}\}\to\left(\min\{\mathcal{O}^{(k)}_{n,i}\}\rightleftarrows\cdots\rightleftarrows\max\{\mathcal{O}^{(k)}_{n,i}\}\right)\leftarrow\min\{\mathcal{N}^{(k)}_{n,i}\}}\times p_{\max\{\mathcal{N}^{(m_{n,i})}_{n,i}\}\to\min\{\mathcal{O}^{(m_{n,i}+1)}_{n,i}\}\to\cdots\to\max\{\mathcal{O}^{(m_{n,i}+1)}_{n,i}\}} \tag{25}$$

where the constituent probabilities can be determined from (21), (23), and (24).

The expression in (25) can be written under the following equivalent form:

$$P^{(\text{IRC2})}_{\mathcal{I}_{n,i}} = p_{\mathcal{I}_{n,i,1}\to(\mathcal{I}_{n,i,1}-1)\to\cdots\to1}\times \prod_{l=1}^{|\mathcal{I}_{n,i}|-1} p_{\mathcal{I}_{n,i,l}\to((\mathcal{I}_{n,i,l}+1)\rightleftarrows\cdots\rightleftarrows(\mathcal{I}_{n,i,l+1}-1))\leftarrow\mathcal{I}_{n,i,l+1}}\times p_{\mathcal{I}_{n,i,|\mathcal{I}_{n,i}|}\to(\mathcal{I}_{n,i,|\mathcal{I}_{n,i}|}+1)\to\cdots\to N} \tag{26}$$

For example, for $N = 8$, $P^{(\text{IRC2})}_{\{2,3,6,7\}} = p_{2\to1}p_{3\to(4\rightleftarrows5)\leftarrow6}p_{7\to8}$ while $P^{(\text{IRC2})}_{\{2,3,5,7\}} = p_{2\to1}p_{3\to4\leftarrow5}p_{5\to6\leftarrow7}p_{7\to8}$.

IV. Diversity Order and Asymptotic Analysis

A. Diversity Order

Consider the outage probability in (14). For large values of the SNR, given that $p_{i,j}$ scales asymptotically as $\mathcal{P}_M^{-\zeta_{i,j}}$

and $q_{i,j} = 1 - p_{i,j} \approx 1$, then the first product in (14) is approximately equal to 1, the second product scales asymptotically as $\mathcal{P}_M^{-\sum_{j'\in\overline{\mathcal{I}}_{n,i}} \zeta_{0,j'}}$, and the third product scales as $\mathcal{P}_M^{-\sum_{j\in\mathcal{I}_{n,i}} \zeta_{j,N+1}}$.

For NIRC, $P^{(\text{IRC})}_{\mathcal{I}_{n,i}} = 1$ in (14) and the diversity order of the NIRC scheme can be written as:

$$\zeta^{(\text{NIRC})} = \zeta_{0,N+1} + \min_{n=0,\dots,N} \min_{i=1,\dots,\binom{N}{n}} \{\zeta^{(0)}_{\mathcal{I}_{n,i}}\} \tag{27}$$

where:

$$\zeta^{(0)}_{\mathcal{I}_{n,i}} \triangleq \sum_{j\in\mathcal{I}_{n,i}} \zeta_{j,N+1} + \sum_{j'\in\overline{\mathcal{I}}_{n,i}} \zeta_{0,j'} \tag{28}$$

For IRC1, the diversity order can be written as:

$$\zeta^{(\text{IRC1})} = \zeta_{0,N+1} + \min_{n=0,\dots,N} \min_{i=1,\dots,\binom{N}{n}} \{\zeta^{(0)}_{\mathcal{I}_{n,i}} + \zeta^{(\text{f})}_{\mathcal{I}_{n,i}}\} \tag{29}$$

where $\zeta^{(\text{f})}_{\mathcal{I}_{n,i}}$ stands for the diversity order of the probability $P^{(\text{IRC1})}_{\mathcal{I}_{n,i}}$ given in (22) where the superscript f stands for the *forward* direction.

In Appendix A we prove that $\zeta^{(\text{f})}_{\mathcal{I}_{n,i}}$ can be written as:

$$\zeta^{(\text{f})}_{\mathcal{I}_{n,i}} = \sum_{l=1}^{|\mathcal{I}_{n,i}|} \zeta_{\mathcal{I}_{n,i,l},\mathcal{I}_{n,i,l}+1} \delta_{\mathcal{I}_{n,i,l}+1\notin\mathcal{I}_{n,i}} \delta_{\mathcal{I}_{n,i,l}\neq N} \tag{30}$$

$$= \sum_{m\in\mathcal{I}_{n,i}\ ;\ m\neq N} \zeta_{m,m+1} \delta_{m+1\notin\mathcal{I}_{n,i}} \tag{31}$$

where $\delta_S = 1$ if the statement S is true and $\delta_S = 0$ otherwise.

For IRC2, the diversity order can be written as:

$$\zeta^{(\text{IRC2})} = \zeta_{0,N+1} + \min_{n=0,\dots,N} \min_{i=1,\dots,\binom{N}{n}} \{\zeta^{(0)}_{\mathcal{I}_{n,i}} + \zeta^{(\text{f,b})}_{\mathcal{I}_{n,i}}\} \tag{32}$$

where $\zeta^{(\text{f,b})}_{\mathcal{I}_{n,i}}$ stands for the diversity order of the probability $P^{(\text{IRC2})}_{\mathcal{I}_{n,i}}$ given in (26) where the superscript b stands for the *backward* direction.

In Appendix B we prove that (32) can be written as:

$$\zeta^{(\text{IRC2})} = \zeta_{0,N+1} + \min_{n=0,\dots,N} \min_{i=1,\dots,\binom{N}{n}} \{\zeta^{(0)}_{\mathcal{I}_{n,i}} + \zeta^{(\text{f})}_{\mathcal{I}_{n,i}} + \zeta^{(\text{b})}_{\mathcal{I}_{n,i}}\} \tag{33}$$

where $\zeta^{(\text{f,b})}_{\mathcal{I}_{n,i}} = \zeta^{(\text{f})}_{\mathcal{I}_{n,i}} + \zeta^{(\text{b})}_{\mathcal{I}_{n,i}}$ and:

$$\zeta^{(\text{b})}_{\mathcal{I}_{n,i}} = \sum_{m\in\mathcal{I}_{n,i}\ ;\ m\neq 1} \zeta_{m,m-1} \delta_{m-1\notin\mathcal{I}_{n,i}} \tag{34}$$

Equations (27), (29), and (33) show that $\zeta^{(\text{IRC2})} \geq \zeta^{(\text{IRC1})} \geq \zeta^{(\text{NIRC})}$. Note that since the parameters $\zeta_{i,j}$ can take arbitrary values depending on the relay positions and misalignment conditions, further simplifications of the expressions in (27), (29), and (33) are not possible in the general case. The diversity orders of NIRC, IRC1, and IRC2 are listed in Table I for $N = 2, 3, 4$ (terms in parentheses $(.)$ correspond to $\zeta^{(0)}_{\mathcal{I}_{n,i}}$ that must be included for NIRC, IRC1, and IRC2; terms in brackets $[.]$ correspond to $\zeta^{(f)}_{\mathcal{I}_{n,i}}$ that must be included for IRC1 and IRC2; terms in braces $\{.\}$ correspond to $\zeta^{(b)}_{\mathcal{I}_{n,i}}$ that must be included for IRC2).

B. Comparison of the IRC schemes with NIRC

Equations (27), (29), and (33) can be written as:

$$\zeta = \zeta_{0,N+1} + \min_{\mathcal{I}\subset\{1,\dots,N\}} \{\zeta^{(0)}_{\mathcal{I}} + \zeta^{(1)}_{\mathcal{I}}\} \tag{35}$$

where $\zeta^{(0)}_{\mathcal{I}}$ is defined in (28). $\zeta^{(1)}_{\mathcal{I}}$ is equal to 0, $\zeta^{(\text{f})}_{\mathcal{I}}$, and $\zeta^{(\text{f})}_{\mathcal{I}} + \zeta^{(\text{b})}_{\mathcal{I}}$ for NIRC, IRC1, and IRC2, respectively, where $\zeta^{(\text{f})}_{\mathcal{I}}$ and $\zeta^{(\text{b})}_{\mathcal{I}}$ are defined in (31) and (34).

Given the cumbersome expressions of the diversity order, it is of extreme importance to highlight under which network conditions will inter-relay cooperation be useful. In [23], it was proven that (27)-(28) can be written as: $\zeta^{(\text{NIRC})} = \zeta_{0,N+1} + \sum_{n=1}^{N} \min\{\zeta_{0,n}, \zeta_{n,N+1}\}$.

1) Case A: Assume first that there are no relays R_n for which $\zeta_{0,n} = \zeta_{n,N+1}$. Construct the set $\mathcal{S}$ as follows:

$$\mathcal{S} = \{n \mid \zeta_{n,N+1} < \zeta_{0,n}\} \tag{36}$$

Therefore, for the considered network, the diversity order that can be achieved by NIRC is given by:

$$\zeta^{(\text{NIRC})} = \zeta_{0,N+1} + \sum_{n\in\mathcal{S}} \zeta_{n,N+1} + \sum_{n'\in\overline{\mathcal{S}}} \zeta_{0,n'} = \zeta_{0,N+1} + \zeta^{(0)}_{\mathcal{S}} \tag{37}$$

where $\min_{\mathcal{I}\subset\{1,\dots,N\}} \zeta^{(0)}_{\mathcal{I}} = \zeta^{(0)}_{\mathcal{S}}$. The condition $\zeta_{0,n} \neq \zeta_{n,N+1}$ for $n = 1,\dots,N$ implies that $\zeta^{(0)}_{\mathcal{S}}$ is the unique minimum and, hence, $\zeta^{(0)}_{\mathcal{I}} > \zeta^{(0)}_{\mathcal{S}}$ for any $\mathcal{I} \neq \mathcal{S}$.

From (35), the diversity order with IRC can be written as:

$$\zeta^{(\text{IRC})} = \zeta_{0,N+1} + \min\left\{\zeta^{(0)}_{\mathcal{S}} + \zeta^{(1)}_{\mathcal{S}}, \min_{\mathcal{I}\neq\mathcal{S}}\{\zeta^{(0)}_{\mathcal{I}} + \zeta^{(1)}_{\mathcal{I}}\}\right\} \tag{38}$$

If $\zeta^{(1)}_{\mathcal{S}} = 0$, then $\min\left\{\zeta^{(0)}_{\mathcal{S}}, \min_{\mathcal{I}\neq\mathcal{S}}\{\zeta^{(0)}_{\mathcal{I}} + \zeta^{(1)}_{\mathcal{I}}\}\right\} = \zeta^{(0)}_{\mathcal{S}}$ since $\zeta^{(0)}_{\mathcal{I}} + \zeta^{(1)}_{\mathcal{I}} \geq \zeta^{(0)}_{\mathcal{I}} > \zeta^{(0)}_{\mathcal{S}}$ resulting in $\zeta^{(\text{IRC})} = \zeta^{(\text{NIRC})}$ and, consequently, the IRC schemes do not result in any enhancement in the diversity order in this case. If $\zeta^{(1)}_{\mathcal{S}} > 0$, then $\zeta^{(0)}_{\mathcal{S}} + \zeta^{(1)}_{\mathcal{S}} > \zeta^{(0)}_{\mathcal{S}}$ and $\zeta^{(0)}_{\mathcal{I}} + \zeta^{(1)}_{\mathcal{I}} \geq \zeta^{(0)}_{\mathcal{I}} > \zeta^{(0)}_{\mathcal{S}}$ resulting in $\min\left\{\zeta^{(0)}_{\mathcal{S}} + \zeta^{(1)}_{\mathcal{S}}, \min_{\mathcal{I}\neq\mathcal{S}}\{\zeta^{(0)}_{\mathcal{I}} + \zeta^{(1)}_{\mathcal{I}}\}\right\} > \zeta^{(0)}_{\mathcal{S}}$ and, consequently, $\zeta^{(\text{IRC})} > \zeta^{(\text{NIRC})}$ implying that inter-relay cooperation is capable of enhancing the diversity order of the network in this case.

Note that, from (31), $\zeta^{(\text{f})}_{\mathcal{S}} = 0$ if there is no element $m \neq N$ of $\mathcal{S}$ for which $m+1$ is not in $\mathcal{S}$. Similarly, from (34), $\zeta^{(\text{b})}_{\mathcal{S}} = 0$ if there is no element $m \neq 1$ of $\mathcal{S}$ for which $m-1$ is not in $\mathcal{S}$. Therefore, for IRC1, $\zeta^{(1)}_{\mathcal{S}} = \zeta^{(\text{f})}_{\mathcal{S}} = 0$ if the first condition is satisfied while for IRC2 $\zeta^{(1)}_{\mathcal{S}} = \zeta^{(\text{f})}_{\mathcal{S}} + \zeta^{(\text{b})}_{\mathcal{S}} = 0$ if the above two conditions are satisfied.

As a conclusion, the usefulness or not of IRC1 and IRC2 with respect to NIRC can be easily revealed by inspecting the set $\mathcal{S}$. In particular, one of the following cases might arise:

- Case 1: $\nexists\ m \in \mathcal{S}\backslash\{N\} \mid m+1 \notin \mathcal{S}$ and $\nexists\ m \in \mathcal{S}\backslash\{1\} \mid m-1 \notin \mathcal{S}$. In this case, $\zeta^{(\text{IRC1})} = \zeta^{(\text{NIRC})}$ and $\zeta^{(\text{IRC2})} = \zeta^{(\text{NIRC})}$ resulting in:

$$\zeta^{(\text{IRC2})} = \zeta^{(\text{IRC1})} = \zeta^{(\text{NIRC})} \tag{39}$$

  implying that there is no additional gain that results from exploiting the R-R links. Note that this case arises only if $\mathcal{S} = \phi$ or $\mathcal{S} = \{1,\dots,N\}$.

TABLE I
DIVERSITY ORDER WITH N RELAYS

N	Diversity Order $-\zeta_{0,N+1}$
2	$\min\{(\zeta_{0,1}+\zeta_{0,2}),(\zeta_{1,3}+\zeta_{0,2})+[\zeta_{1,2}],(\zeta_{2,3}+\zeta_{0,1})+\{\zeta_{2,1}\},(\zeta_{1,3}+\zeta_{2,3})\}$
3	$\min\{(\zeta_{0,1}+\zeta_{0,2}+\zeta_{0,3}),(\zeta_{1,4}+\zeta_{0,2}+\zeta_{0,3})+[\zeta_{1,2}],(\zeta_{2,4}+\zeta_{0,1}+\zeta_{0,3})+[\zeta_{2,3}]+\{\zeta_{2,1}\},(\zeta_{3,4}+\zeta_{0,1}+\zeta_{0,2})+\{\zeta_{3,2}\},(\zeta_{1,4}+\zeta_{2,4}+\zeta_{0,3})+[\zeta_{2,3}],$ $(\zeta_{1,4}+\zeta_{3,4}+\zeta_{0,2})+[\zeta_{1,2}]+\{\zeta_{3,2}\},(\zeta_{2,4}+\zeta_{3,4}+\zeta_{0,1})+\{\zeta_{2,1}\},(\zeta_{1,4}+\zeta_{2,4}+\zeta_{3,4})\}$
4	$\min\{(\zeta_{0,1}+\zeta_{0,2}+\zeta_{0,3}+\zeta_{0,4}),(\zeta_{1,5}+\zeta_{0,2}+\zeta_{0,3}+\zeta_{0,4})+[\zeta_{1,2}],(\zeta_{2,5}+\zeta_{0,1}+\zeta_{0,3}+\zeta_{0,4})+[\zeta_{2,3}]+\{\zeta_{2,1}\},(\zeta_{3,5}+\zeta_{0,1}+\zeta_{0,2}+\zeta_{0,4})+[\zeta_{3,4}]+\{\zeta_{3,2}\},$ $(\zeta_{4,5}+\zeta_{0,1}+\zeta_{0,2}+\zeta_{0,3})+\{\zeta_{4,3}\},(\zeta_{1,5}+\zeta_{2,5}+\zeta_{0,3}+\zeta_{0,4})+[\zeta_{2,3}],(\zeta_{1,5}+\zeta_{3,5}+\zeta_{0,2}+\zeta_{0,4})+[\zeta_{1,2}+\zeta_{3,4}]+\{\zeta_{3,2}\},$ $(\zeta_{1,5}+\zeta_{4,5}+\zeta_{0,2}+\zeta_{0,3})+[\zeta_{1,2}]+\{\zeta_{4,3}\},(\zeta_{2,5}+\zeta_{3,5}+\zeta_{0,1}+\zeta_{0,4})+[\zeta_{3,4}]+\{\zeta_{2,1}\},(\zeta_{2,5}+\zeta_{4,5}+\zeta_{0,1}+\zeta_{0,3})+[\zeta_{2,3}]+\{\zeta_{2,1}+\zeta_{4,3}\},$ $(\zeta_{3,5}+\zeta_{4,5}+\zeta_{0,1}+\zeta_{0,2})+\{\zeta_{3,2}\},(\zeta_{1,5}+\zeta_{2,5}+\zeta_{3,5}+\zeta_{0,4})+[\zeta_{3,4}],(\zeta_{1,5}+\zeta_{2,5}+\zeta_{4,5}+\zeta_{0,3})+[\zeta_{2,3}]+\{\zeta_{4,3}\},$ $(\zeta_{1,5}+\zeta_{3,5}+\zeta_{4,5}+\zeta_{0,2})+[\zeta_{1,2}]+\{\zeta_{3,2}\},(\zeta_{2,5}+\zeta_{3,5}+\zeta_{4,5}+\zeta_{0,1})+\{\zeta_{2,1}\},(\zeta_{1,5}+\zeta_{2,5}+\zeta_{3,5}+\zeta_{4,5})\}$

- Case 2: $\nexists\ m\in\mathcal{S}\backslash\{N\}\mid m+1\notin\mathcal{S}$ and $\exists\ m\in\mathcal{S}\backslash\{1\}\mid m-1\notin\mathcal{S}$. In this case, $\zeta^{(\text{IRC1})}=\zeta^{(\text{NIRC})}$ and $\zeta^{(\text{IRC2})}>\zeta^{(\text{NIRC})}$ resulting in:

$$\zeta^{(\text{IRC2})}>\zeta^{(\text{IRC1})}=\zeta^{(\text{NIRC})} \tag{40}$$

  implying that only the two-way scheme IRC2 can result in a diversity gain in this case.
- Case 3: $\exists\,m\in\mathcal{S}\backslash\{N\}\mid m+1\notin\mathcal{S}$. In this case, $\zeta^{(\text{IRC1})}>\zeta^{(\text{NIRC})}$ and $\zeta^{(\text{IRC2})}>\zeta^{(\text{NIRC})}$ resulting in:

$$\zeta^{(\text{IRC2})}\geq\zeta^{(\text{IRC1})}>\zeta^{(\text{NIRC})} \tag{41}$$

  and inter-relay cooperation is capable of boosting the diversity order of the network. In this case, while both IRC1 and IRC2 outperform NIRC, it is not possible to determine if IRC2 is capable of outperforming IRC1 or not. In fact, whether $\zeta^{(\text{IRC2})}=\zeta^{(\text{IRC1})}$ or $\zeta^{(\text{IRC2})}>\zeta^{(\text{IRC1})}$ depends on other parameters of the network and not only on the set $\mathcal{S}$ as in the case of comparing IRC1 and IRC2 with NIRC as will be highlighted later.

Interestingly, the structure of the set $\mathcal{S}$ depends on the states of the S-R and R-D links but not on the states of the R-R links. For example, consider a 3-relay network. (i): If $\zeta_{1,4}>\zeta_{0,1}$, $\zeta_{2,4}>\zeta_{0,2}$, and $\zeta_{3,4}<\zeta_{0,3}$, then $\mathcal{S}=\{3\}$ resulting in case 2 above since $3\in\mathcal{S}$ while $3-1=2\notin\mathcal{S}$. Therefore, IRC2 is recommended for this network. (ii): If $\zeta_{1,4}<\zeta_{0,1}$, $\zeta_{2,4}<\zeta_{0,2}$, and $\zeta_{3,4}>\zeta_{0,3}$, then $\mathcal{S}=\{1,2\}$ resulting in case 3 above since $2\in\mathcal{S}$ while $2+1=3\notin\mathcal{S}$. Therefore, at this level, both IRC1 and IRC2 constitute valid choices for this network

Finally, it is worth noting that for negligible misalignment fading, $\mathcal{S}=\{n\mid d_{n,N+1}>d_{0,n}\}$ since $\zeta_{i,j}=\beta_{i,j}$ that decreases with the distance $d_{i,j}$. Consequently, the usefulness or not of IRC can be deduced from the geometry of the network.

2) Case B: Consider now the case where the relation $\zeta_{0,n}=\zeta_{n,N+1}$ holds for some relays where the set containing the indices of these relays will be denoted by $\mathcal{S}^{(\text{eq})}$. In this case, (37) can be written under the following form:

$$\zeta^{(\text{NIRC})}=\zeta_{0,N+1}+\sum_{n\in\mathcal{S}}\zeta_{n,N+1}+\sum_{n'\in\overline{\mathcal{S}}\backslash\mathcal{S}^{(\text{eq})}}\zeta_{0,n'}+\sum_{n''\in\mathcal{S}^{(\text{eq})}}\zeta_{n'',N+1} \tag{42}$$

where the summands of the last summation can also be written as $\zeta_{0,n''}$. Now, elements can be moved from the third summation to either one of the first two summations without changing the value of (42) since $\zeta_{0,n''}=\zeta_{n'',N+1}$. In other words, moving the elements of any subset $\mathcal{S}^{(\text{eq})}_{\text{sub}}$ of $\mathcal{S}^{(\text{eq})}$ from the third summation to the first summation while moving the elements of $\mathcal{S}^{(\text{eq})}\backslash\mathcal{S}^{(\text{eq})}_{\text{sub}}$ from the third summation to the second summation, (42) can be written as:

$$\zeta^{(\text{NIRC})}=\zeta_{0,N+1}+\sum_{n\in\mathcal{S}\cup\mathcal{S}^{(\text{eq})}_{\text{sub}}}\zeta_{n,N+1}+\sum_{n'\in\overline{\mathcal{S}\cup\mathcal{S}^{(\text{eq})}_{\text{sub}}}}\zeta_{0,n'} \tag{43}$$

$$=\zeta_{0,N+1}+\zeta^{(0)}_{\mathcal{S}\cup\mathcal{S}^{(\text{eq})}_{\text{sub}}} \tag{44}$$

In other words, the set $\{\zeta^{(0)}_{\mathcal{I}}\}_{\mathcal{I}\subset\{1,\dots,N\}}$ of 2^N elements contains $2^{|\mathcal{S}^{(\text{eq})}|}$ elements that assume the same minimum value where each one of these elements corresponds to a possible set $\mathcal{S}\cup\mathcal{S}^{(\text{eq})}_{\text{sub}}$. In this case, the IRC schemes can not improve the diversity order unless all of these minima are increased. In other words, the relation $\min_{\mathcal{I}\subset\{1,\dots,N\}}\zeta^{(0)}_{\mathcal{I}}=\zeta^{(0)}_{\mathcal{S}}$ in the case of no relays satisfying $\zeta_{0,n}=\zeta_{n,N+1}$ (case A) needs to be extended to $\min_{\mathcal{I}\subset\{1,\dots,N\}}\zeta^{(0)}_{\mathcal{I}}=\zeta^{(0)}_{\mathcal{S}\cup\mathcal{S}^{(\text{eq})}_{\text{sub}}}$ for any subset $\mathcal{S}^{(\text{eq})}_{\text{sub}}$ of $\mathcal{S}^{(\text{eq})}$. Moreover, the relation $\zeta^{(0)}_{\mathcal{I}}>\zeta^{(0)}_{\mathcal{S}}$ for any $\mathcal{I}\neq\mathcal{S}$ does not hold as in case A since there are $2^{|\mathcal{S}^{(\text{eq})}|}-1$ additional sets for which $\zeta^{(0)}_{\mathcal{I}}=\zeta^{(0)}_{\mathcal{S}}$. All of these sets must be taken into consideration to determine under which one of the scenarios the network falls.

Define the following two true-false functions:

$$\mathfrak{F}_1(\mathcal{I})=\begin{cases}1, & \exists\ m\in\mathcal{I}\backslash\{N\}\mid m+1\notin\mathcal{I};\\ 0, & \text{otherwise.}\end{cases} \tag{45}$$

$$\mathfrak{F}_2(\mathcal{I})=\begin{cases}1, & \exists\ m\in\mathcal{I}\backslash\{1\}\mid m-1\notin\mathcal{I};\\ 0, & \text{otherwise.}\end{cases} \tag{46}$$

The usefulness or not of IRC1 and IRC2 can now be established by inspecting the union of the set $\mathcal{S}$ with all possible subsets of $\mathcal{S}^{(\text{eq})}$. Let $f_k\triangleq\prod_{\mathcal{S}^{(\text{eq})}_{\text{sub}}\subset\mathcal{S}^{(\text{eq})}}\mathfrak{F}_k(\mathcal{S}\cup\mathcal{S}^{(\text{eq})}_{\text{sub}})$ for $k=1,2$.

- If $f_1=0$ and $f_2=0$, $\zeta^{(\text{IRC2})}=\zeta^{(\text{IRC1})}=\zeta^{(\text{NIRC})}$ corresponding to case 1 above.
- If $f_1=0$ and $f_2=1$, $\zeta^{(\text{IRC2})}>\zeta^{(\text{IRC1})}=\zeta^{(\text{NIRC})}$ corresponding to case 2 above.
- If $f_1=1$, $\zeta^{(\text{IRC2})}\geq\zeta^{(\text{IRC1})}>\zeta^{(\text{NIRC})}$ corresponding to case 3 above.

For example, consider a 4-relay network. If $\zeta_{1,5}=\zeta_{0,1}$, $\zeta_{2,5}>\zeta_{0,2}$, $\zeta_{3,5}=\zeta_{0,3}$ and $\zeta_{4,5}<\zeta_{0,4}$, then $\mathcal{S}=\{4\}$ and $\mathcal{S}^{(\text{eq})}=\{1,3\}$ resulting in $\mathcal{S}^{(\text{eq})}_{\text{sub}}$ being ϕ, $\{1\}$, $\{3\}$ or $\{1,3\}$. Let $\mathcal{S}_1\triangleq\{4\}$, $\mathcal{S}_2\triangleq\{1,4\}$, $\mathcal{S}_3\triangleq\{3,4\}$, and $\mathcal{S}_4\triangleq\{1,3,4\}$. In this case, $\mathfrak{F}_1(\mathcal{S}_1)=0$ implying directly that $f_1=0$. On the other hand, $\mathfrak{F}_2(\mathcal{S}_1)=\mathfrak{F}_2(\mathcal{S}_2)=\mathfrak{F}_2(\mathcal{S}_3)=\mathfrak{F}_2(\mathcal{S}_4)=1$ resulting in $f_2=1$. Therefore, $\zeta^{(\text{IRC2})}>\zeta^{(\text{IRC1})}=\zeta^{(\text{NIRC})}$ showing that IRC2 is the best solution for this network.

Note that under weak misalignment fading, the relation $\zeta_{0,n} = \zeta_{n,N+1}$ translates into $\beta_{0,n} = \beta_{n,N+1}$ and $d_{0,n} = d_{n,N+1}$ implying that R_n is in the median plane. An interesting special case arises when all relays are in the median plane. In this case, $\mathcal{S} = \phi$ while $\mathcal{S}^{(\text{eq})} = \{1,\dots,N\}$. Considering $\mathcal{S}^{(\text{eq})}_{\text{sub}} = \mathcal{S}^{(\text{eq})}$ results in the set $\mathcal{S} \cup \mathcal{S}^{(\text{eq})}_{\text{sub}} = \{1,\dots,N\}$ for which $\mathfrak{F}_1(\{1,\dots,N\}) = \mathfrak{F}_2(\{1,\dots,N\}) = 0$ resulting in $f_1 = f_2 = 0$ irrespective of the values $\mathfrak{F}_1(.)$ and $\mathfrak{F}_2(.)$ yielded by the other subsets. This results in the important conclusion that the IRC solutions can not improve the diversity order of an FSO network for which all relays belong to the median plane, or equivalently, for which $\zeta_{0,n} = \zeta_{n,N+1}$ for $n = 1,\dots,N$ under any misalignment conditions. Similarly, IRC will not be privileged if more relays are in the median plane since the number of elements in $\mathcal{S}^{(\text{eq})}$ will increase decreasing the chances of f_1 and f_2 to be 1.

C. Comparison between IRC1 with IRC2 in Case 3

It is more convenient to tackle the problem by analyzing the diversity gain $\Delta\zeta^{(\text{IRC})} \triangleq \zeta^{(\text{IRC})} - \zeta^{(\text{NIRC})}$ of an IRC scheme with respect to the NIRC scheme. From (35), given that $\zeta^{(\text{NIRC})} = \zeta_{0,N+1} + \zeta^{(0)}_{\mathcal{S}}$, then $\Delta\zeta^{(\text{IRC})} = \min_{\mathcal{I}\subset\{1,\dots,N\}}\{\zeta^{(0)}_{\mathcal{I}} - \zeta^{(0)}_{\mathcal{S}} + \zeta^{(1)}_{\mathcal{I}}\}$. It can be easily proven that $\zeta^{(0)}_{\mathcal{I}} - \zeta^{(0)}_{\mathcal{S}} = \sum_{n\in\mathcal{I}\oplus\mathcal{S}} \chi_n$ where $\mathcal{I}\oplus\mathcal{S}$ stands for the set of elements that belong to $\mathcal{I}\cup\mathcal{S}$ but not to $\mathcal{I}\cap\mathcal{S}$ and:

$$\chi_n \triangleq |\zeta_{0,n} - \zeta_{n,N+1}| \tag{47}$$

Therefore, the diversity gain can be written as:

$$\Delta\zeta^{(\text{IRC})} = \min_{\mathcal{I}\subset\{1,\dots,N\}}\{\Delta\zeta^{(0)}_{\mathcal{I}} + \zeta^{(1)}_{\mathcal{I}}\}\ ;\ \Delta\zeta^{(0)}_{\mathcal{I}} \triangleq \sum_{n\in\mathcal{I}\oplus\mathcal{S}} \chi_n \tag{48}$$

We will start with an illustrative example that sheds more light on this case. Assume that $N = 3$ and $\mathcal{S} = \{2\}$. Considering all possible subsets $\mathcal{I}$ of $\{1,\dots,N\}$ and after some simplifications, the diversity gains of IRC1 and IRC2 can be written as:

$$\Delta\zeta^{(\text{IRC1})} = \min\{\chi_2, \chi_3, \zeta_{2,3}\} \tag{49}$$

$$\Delta\zeta^{(\text{IRC2})} = \min\{\chi_2, \chi_1+\chi_3, \chi_1+\zeta_{2,3}, \chi_3+\zeta_{2,1}, \zeta_{2,3}+\zeta_{2,1}\} \tag{50}$$

Consider now the following scenarios. (i): If $\chi_2 < \chi_3$ and $\chi_2 < \zeta_{2,3}$, then $\Delta\zeta^{(\text{IRC1})} = \chi_2$ and $\Delta\zeta^{(\text{IRC2})} = \chi_2$ resulting in $\Delta\zeta^{(\text{IRC2})} = \Delta\zeta^{(\text{IRC1})}$ implying that IRC2 does not present any diversity advantage over IRC1 in this case. (ii): If $\chi_3 < \chi_2$ and $\chi_3 < \zeta_{2,3}$, then $\Delta\zeta^{(\text{IRC1})} = \chi_3$ while $\Delta\zeta^{(\text{IRC2})} > \chi_3$ since $\chi_2 > \chi_3$, $\chi_1+\chi_3 > \chi_3$, $\chi_1+\zeta_{2,3} > \zeta_{2,3} > \chi_3$, $\chi_3+\zeta_{2,1} > \chi_3$ and $\zeta_{2,3}+\zeta_{2,1} > \zeta_{2,3} > \chi_3$. Consequently, $\Delta\zeta^{(\text{IRC2})} > \Delta\zeta^{(\text{IRC1})}$ and IRC2 results in a higher diversity order in this case. (iii): If $\zeta_{2,3} < \chi_2$ and $\zeta_{2,3} < \chi_3$, then $\Delta\zeta^{(\text{IRC1})} = \zeta_{2,3}$ while $\Delta\zeta^{(\text{IRC2})} > \zeta_{2,3}$ since $\chi_2 > \zeta_{2,3}$, $\chi_1+\chi_3 > \chi_3 > \zeta_{2,3}$, $\chi_1+\zeta_{2,3} > \zeta_{2,3}$, $\chi_3+\zeta_{2,1} > \chi_3 > \zeta_{2,3}$, and $\zeta_{2,3}+\zeta_{2,1} > \zeta_{2,3}$. Therefore, $\Delta\zeta^{(\text{IRC2})}$ is strictly greater than $\Delta\zeta^{(\text{IRC1})}$ in this case as well. As a conclusion:

$$\begin{cases} \Delta\zeta^{(\text{IRC2})} = \Delta\zeta^{(\text{IRC1})}, & \min\{\chi_2,\chi_3,\zeta_{2,3}\} = \chi_2; \\ \Delta\zeta^{(\text{IRC2})} > \Delta\zeta^{(\text{IRC1})}, & \text{otherwise.} \end{cases} \tag{51}$$

and, hence, the comparison between IRC1 and IRC2 depends not only on the set $\mathcal{S}$ but also on the values of χ_2, χ_3, and $\zeta_{2,3}$.

On the other hand, if $\mathcal{S} = \{1\}$, it can be proven in a similar way that $\Delta\zeta^{(\text{IRC2})} = \Delta\zeta^{(\text{IRC1})} = \min\{\chi_1, \chi_2+\chi_3, \chi_2+\zeta_{2,3}, \zeta_{1,2}\}$ for all values of χ_1, χ_2, χ_3, $\zeta_{1,2}$, and $\zeta_{2,3}$.

Carrying out the comparison between IRC1 and IRC2 for any N turns out to be tedious where the results are highly dependent on the particular value of the set $\mathcal{S}$ making it hard to reach a closed-form generic solution that covers all possible cases. Consequently, we will resort to an assumption that simplifies the comparison of the above IRC schemes.

Equation (48) shows that $\Delta\zeta^{(\text{IRC})}$ depends on the parameters $\{\chi_n\}_{n=1}^{N}$ (through the term $\Delta\zeta^{(0)}_{\mathcal{I}}$) and $\{\zeta_{n,n+1}\}_{n=1}^{N-1}$ (through the term $\zeta^{(1)}_{\mathcal{I}}$). Since the parameters χ correspond mainly to the difference between two ζ parameters, then it is appropriate to assume that the former parameters are smaller than the latter ones. This is especially true if (i): the S-R_n and R_n-D links are of comparable distances and manifest similar misalignment conditions (resulting in small values of χ_n) and/or (ii): the inter-relay links are short with weak misalignment fading (resulting in large values of $\zeta_{n,n+1}$). Therefore, the comparison will be performed under the assumption:

$$\chi_{n'} = |\zeta_{0,n'} - \zeta_{n',N+1}| \ll \zeta_{n,n+1}\ \forall\ n, n' \tag{52}$$

For IRC1, it can be observed that $\zeta^{(1)}_{\mathcal{I}} \neq 0$ for all 2^N subsets $\mathcal{I}$ of $\{1,\dots,N\}$ except for the following $N+1$ sets: ϕ, $\{1,\dots,N\}$, $\{2,\dots,N\},\dots,\{N\}$. Consequently, based on the assumption in (52) and since $\zeta^{(1)}_{\mathcal{I}}$ contains terms of the form $\zeta_{n,n+1}$, then the diversity gain of IRC1 can be written as $\Delta\zeta^{(\text{IRC1})} = \min_{\mathcal{I}}\{\Delta\zeta^{(0)}_{\mathcal{I}}\}$ where the minimization is limited over the above $N+1$ sets. For IRC2, $\zeta^{(1)}_{\mathcal{I}} \neq 0$ for all subsets of $\{1,\dots,N\}$ except for the sets ϕ and $\{1,\dots,N\}$ resulting in $\Delta\zeta^{(\text{IRC2})} = \min\{\Delta\zeta^{(0)}_{\phi}, \Delta\zeta^{(0)}_{\{1,\dots,N\}}\}$.

Therefore, the comparison between IRC1 and IRC2 can be performed in a simple way as follows: if the set $\mathcal{I}$ that minimizes $\Delta\zeta^{(\text{IRC1})}$ is either ϕ or $\{1,\dots,N\}$, then $\Delta\zeta^{(\text{IRC2})} = \Delta\zeta^{(\text{IRC1})}$; otherwise, $\Delta\zeta^{(\text{IRC2})} > \Delta\zeta^{(\text{IRC1})}$. For example, for $N = 3$ and $\mathcal{S} = \{2\}$: $\Delta\zeta^{(0)}_{\phi} = \sum_{n\in\phi\oplus\mathcal{S}=\{2\}} = \chi_2$, $\Delta\zeta^{(0)}_{\{1,2,3\}} = \sum_{n\in\{1,2,3\}\oplus\mathcal{S}=\{1,3\}} = \chi_1+\chi_3$, $\Delta\zeta^{(0)}_{\{2,3\}} = \sum_{n\in\{2,3\}\oplus\mathcal{S}=\{3\}} = \chi_3$, and $\Delta\zeta^{(0)}_{\{3\}} = \sum_{n\in\{3\}\oplus\mathcal{S}=\{2,3\}} = \chi_2+\chi_3$. In this case, $\Delta\zeta^{(\text{IRC1})} = \min\{\chi_2, \chi_1+\chi_3, \chi_3, \chi_2+\chi_3\} = \min\{\chi_2,\chi_3\}$. If $\chi_2 < \chi_3$, then the set that minimizes $\Delta\zeta^{(\text{IRC1})}$ is $\mathcal{I} = \phi$ implying that $\Delta\zeta^{(\text{IRC2})} = \Delta\zeta^{(\text{IRC1})}$. On the other hand, if $\chi_3 < \chi_2$, then the set that minimizes $\Delta\zeta^{(\text{IRC1})}$ is $\mathcal{I} = \{2,3\}$ implying that $\Delta\zeta^{(\text{IRC2})} > \Delta\zeta^{(\text{IRC1})}$. This result is coherent with (51) if $\zeta_{2,3}$ is large so that $\min\{\chi_2,\chi_3,\zeta_{2,3}\} = \min\{\chi_2,\chi_3\}$. On the other hand, for $\mathcal{S} = \{1\}$, $\Delta\zeta^{(0)}_{\phi} = \chi_1$, $\Delta\zeta^{(0)}_{\{1,2,3\}} = \chi_2+\chi_3$, $\Delta\zeta^{(0)}_{\{2,3\}} = \chi_1+\chi_2+\chi_3$, and $\Delta\zeta^{(0)}_{\{3\}} = \chi_1+\chi_3$ resulting in $\Delta\zeta^{(\text{IRC1})} = \min\{\chi_1, \chi_2+\chi_3\}$. In this case, $\Delta\zeta^{(\text{IRC1})}$ is minimized with either ϕ or $\{1,2,3\}$ implying that $\Delta\zeta^{(\text{IRC2})}$ is always equal to $\Delta\zeta^{(\text{IRC1})}$ for this network in coherence with the previously provided direct comparison.

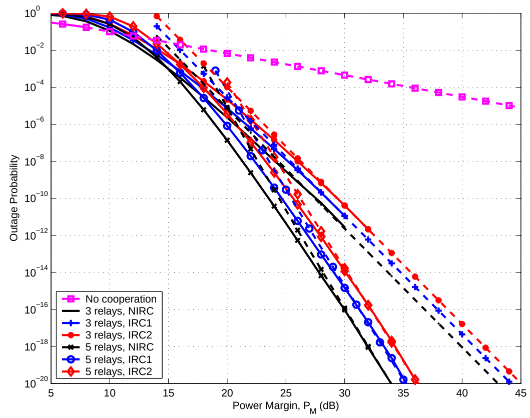

Fig. 2. Performance under scenario 1. Solid and dashed lines correspond to the exact and approximate outage probabilities.

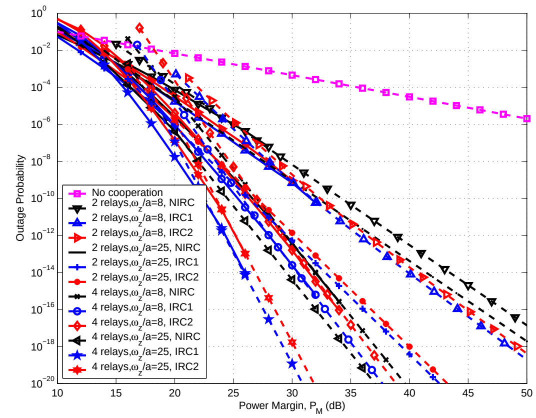

Fig. 3. Performance under scenario 2. Solid and dashed lines correspond to the exact and approximate outage probabilities.

V. Numerical Results

The refractive index structure constant and the attenuation constant are set to $C_n^2 = 1\times10^{-14}$ $\text{m}^{-2/3}$ and $\sigma = 0.44$ dB/km. In all scenarios, the distance between S and D is $d_{0,N+1} = 5$ km. The receiver radius, beam waist, and pointing error displacement standard deviation are assumed to be the same for all links and they will be denoted by a, ω_z, and σ_s, respectively. In what follows, we set $\sigma_s/a = 3$. The values of ω_z/a will be varied in the simulations where large values of this ratio indicate less pointing errors. The set of distances $\mathcal{D}$ is defined as: $\mathcal{D} \triangleq \{\pm(d_{0,n}, d_{n,N+1})\}_{n=1}^{N}$ where the sign $+$ (resp. $-$) indicates that the relay is above (resp. below) the line formed by joining S and D in a two-dimensional plane. We will provide simulations under different network configurations reflecting the following four scenarios that might arise when comparing the IRC and NIRC schemes. Scenario 1: $\zeta^{(\text{IRC2})} = \zeta^{(\text{IRC1})} = \zeta^{(\text{NIRC})}$, scenario 2: $\zeta^{(\text{IRC2})} > \zeta^{(\text{IRC1})} = \zeta^{(\text{NIRC})}$, scenario 3: $\zeta^{(\text{IRC2})} = \zeta^{(\text{IRC1})} > \zeta^{(\text{NIRC})}$ and scenario 4: $\zeta^{(\text{IRC2})} > \zeta^{(\text{IRC1})} > \zeta^{(\text{NIRC})}$. An extensive simulation campaign highlighted the extremely close match between the numerical and analytical results (where the corresponding curves were barely distinguishable) thus supporting the validity of the provided derivations.

Fig. 2 shows the performance of 3-relay and 5-relay networks for which neither IRC1 nor IRC2 is useful corresponding to scenario 1. We set $\omega_z/a = 10$, $\mathcal{D} = \{(1, 4.2), (1.5, 3.6), -(2, 3.1)\}$ for $N = 3$ and $\mathcal{D} = \{(3, 2.4), (3.3, 2), (3.6, 1.6), -(3.7, 1.5), -(3.8, 2.2)\}$ for $N = 5$. This scenario corresponds to case 1 in subsection IV-B where $\mathcal{S} = \{1, 2, 3\}$ for $N = 3$ and $\mathcal{S} = \phi$ for $N = 5$. Results show the very close match between the exact outage probabilities based on (8) and the asymptotic values based on (10) for large values of the power margin $\mathcal{P}_M$. Results also support the accuracy of the derived expressions for the diversity order where the analytical values based on (27) and (28) for NIRC, on (29) and (31) for IRC1 and on (31), (33) and (34) for IRC2 closely match the negative slopes of the different outage probability curves. These formulas accurately predict diversity orders of 6.4 and 10 for $N = 3$ and $N = 5$, respectively. For this scenario, NIRC is the best solution not only because it achieves the same diversity order as IRC1 and IRC2 with a reduced system complexity but also since it achieves a slightly better performance than these two IRC schemes. This results from the increase of the total number of links N_{link} from NIRC to IRC1 and IRC2 implying that the transmit power will be divided among a larger number of links.

In Fig. 3 we provide examples of networks with different number of relays for which scenario 2 arises. We set $\omega_z/a = 25$ while $\mathcal{D}$ takes the following values: $\{(2.6, 2.5), (3.2, 1.8), -(2.7, 4.6)\}$ for $N = 3$, $\{(2.6, 2.5), (3.2, 1.8), -(2.7, 4.3), -(2.9, 4.7)\}$ for $N = 4$ and $\{(2.6, 2.5), (3.4, 1.6), -(2.7, 3.4), -(2.6, 3.9), -(2.6, 4.3), -(2.7, 4.6)\}$ for $N = 6$. The superiority of IRC2 over IRC1 (that achieves the same diversity order as NIRC) was predicted theoretically by case 2 in subsection IV-B since $\mathcal{S} = \{3\}$ for $N = 3$, $\mathcal{S} = \{3, 4\}$ for $N = 4$ and $\mathcal{S} = \{3, 4, 5, 6\}$ for $N = 6$. For the considered simulation setup, the gain in the diversity order offered by IRC2 (with respect to either IRC1 or NIRC) is 0.86, 1.46, and 2.93 with 3, 4, and 6 relays, respectively. In all scenarios, the performance gains with respect to non-cooperative systems are huge for average-to-large values of $\mathcal{P}_M$.

Scenario 3 is reflected in Fig. 4 with $N = 2$ and $N = 4$ for $\omega_z/a = 8$ and $\omega_z/a = 25$. We set $\mathcal{D} = \{(1, 4.1), -(4.1, 1)\}$ for $N = 2$ and $\mathcal{D} = \{(1, 4.1), (1.5, 3.5), -(3.2, 1.9), -(4, 1.8)\}$ for $N = 4$. Results highlight the enhanced diversity orders and performance levels that can be achieved by activating the inter-relay links. In this scenario, IRC1 and IRC2 achieve the same diversity order where the outage probability curves are practically parallel to each other for large values of $\mathcal{P}_M$. This renders IRC1 the most adapted solution under this scenario. In this case, IRC2 even results in a small performance loss with respect to IRC1 since the transmit power needs to be divided among a larger number of links. In this example, for $N = 2$, the diversity order of NIRC does not increase when ω_z/a increases from 8 to 25 where the diversity order remains 4.33. This shows that the performance of the NIRC network is limited mainly by atmospheric turbulence rather than pointing

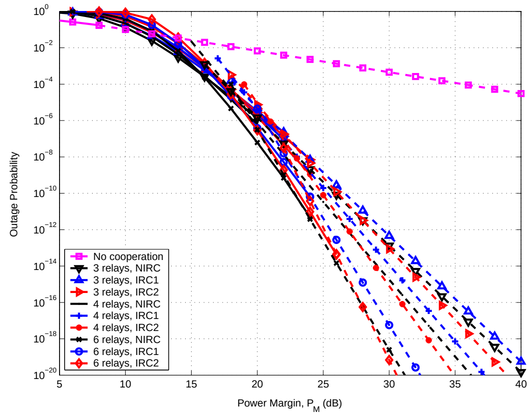

Fig. 4. Performance under scenario 3. Solid and dashed lines correspond to the exact and approximate outage probabilities.

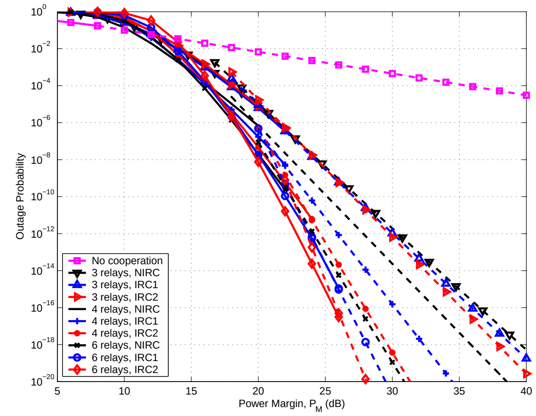

Fig. 5. Performance under scenario 4. Solid and dashed lines correspond to the exact and approximate outage probabilities.

errors; in this case, reducing the pointing errors does not manifest in an improved diversity order. Interestingly, this is not the case with IRC where the diversity order increases from 4.64 for $\omega_z/a = 8$ to 6.15 for $\omega_z/a = 25$. For the IRC network, both atmospheric turbulence and pointing errors affect the performance and, hence, reducing the pointing errors results in an increase in the diversity order. This is reflected in large performance gains that range from 2.5 dB for $\omega_z/a = 8$ to 6 dB for $\omega_z/a = 25$ when comparing IRC1 with NIRC at an outage probability of 10^{-10}. For $N = 4$, $\zeta^{(\text{NIRC})} = 7.88$ and $\zeta^{(\text{IRC2})} = \zeta^{(\text{IRC1})} = 8.16$ for $\omega_z/a = 8$ while $\zeta^{(\text{NIRC})} = 7.94$ and $\zeta^{(\text{IRC2})} = \zeta^{(\text{IRC1})} = 11.96$ for $\omega_z/a = 25$.

Scenario 4 is reflected in Fig. 5 for $\omega_z/a = 25$ with different number of relays. The values of $\mathcal{D}$ are $\{(2.7, 2.2), -(2.8, 4.6), -(3.7, 3.2)\}$ for $N = 3$, $\{(1, 4.1), (4.1, 1), -(3.9, 1.5), -(1.5, 3.9)\}$ for $N = 4$, and $\{(1, 4.1), (4.1, 1), -(3.9, 1.5), -(2.3, 2.9), -(2.9, 2.3), -(1.5, 3.9)\}$ for $N = 6$. The simulated network for $N = 3$ corresponds to the example provided in subsection IV-C where the diversity gains of IRC1 and IRC2 with respect to NIRC are provided in (49)-(50). For this network, $\chi_1 = 0.75$, $\chi_2 = 0.86$, $\chi_3 = 0.23$, $\zeta_{1,2} = 1.94$, and $\zeta_{2,3} = 3.76$ implying that IRC2 will achieve a higher diversity order than IRC1 according to (51). In this case, the diversity order of IRC2 exceeds the diversity order of IRC1 by $\Delta\zeta^{(\text{IRC2})} - \Delta\zeta^{(\text{IRC1})} = \chi_2 - \chi_3 = 0.63$. For $N = 4$, huge gains in the diversity order can be observed where $\zeta^{(\text{NIRC})} = 7.46$, $\zeta^{(\text{IRC1})} = 9.34$, and $\zeta^{(\text{IRC2})} = 11.7$. Similarly, $\zeta^{(\text{NIRC})} = 11.58$, $\zeta^{(\text{IRC1})} = 14.13$, and $\zeta^{(\text{IRC2})} = 17.7$ for $N = 6$.

VI. Conclusion

In the context of FSO collaborative systems, communicating over the existing relay-relay links constitutes an additional degree of freedom that can be exploited to enhance the achievable diversity orders and performance levels. Special consideration needs to be paid to the engineering of such systems since inter-relay cooperation is not useful in all circumstances. Even in the scenarios where inter-relay cooperation is capable of increasing the diversity order, the achievable gains are highly dependent on the particular network topology. In some cases, the minor gains in the diversity order do not justify the upsurge in the system complexity that results from implementing the IRC techniques; in other cases, significant gains can be reached stressing on the huge potential of IRC techniques.

Appendix A

Consider the probability in (22) and let $u = \mathcal{I}_{n,i,l}$ and $v = \mathcal{I}_{n,i,l+1}$. The following cases arise:

(i): $v = u + 1 \Rightarrow p_{u \to \cdots \to v-1} = p_{u \to u} = 1$ and the corresponding diversity order is zero.

(ii): $v > u + 2$; in this case, $p_{u \to \cdots \to v-1} = p_{u,u+1} + q_{u,u+1} p_{u+1,N+1} p_{u+1 \to \cdots \to v-1}$ where $p_{u+1 \to \cdots \to v-1}$ in this case can be written as the sum of different terms where each one of these terms is either of the form $p_{i,j}$ or corresponds to the product of two or more probabilities of the form $p_{i,j}$. Consequently, the probability $q_{u,u+1} p_{u+1,N+1} p_{u+1 \to \cdots \to v-1}$ is equal to the summation of different terms where each term corresponds to the product of two or more probabilities of the form $p_{i,j}$; therefore, $q_{u,u+1} p_{u+1,N+1} p_{u+1 \to \cdots \to v-1}$ is several orders of magnitude smaller than $p_{u,u+1}$ and hence $p_{u \to \cdots \to v-1} \approx p_{u,u+1}$ and the corresponding diversity order is $\zeta_{u,u+1}$.

(iii): $v = u + 2$; in this case, the corresponding probability can be written as $p_{u \to \cdots \to v-1} = p_{u \to u+1} = p_{u,u+1} + q_{u,u+1} p_{u+1,N+1}$ that scales asymptotically as $\mathcal{P}_M^{-\min\{\zeta_{u,u+1}, \zeta_{u+1,N+1}\}}$ given that $q_{u,u+1} \approx 1$. In what follows, we prove that when taking the sets other than $\mathcal{I}_{n,i}$ into consideration, the dominant probability in $p_{u \to u+1}$ is always $p_{u,u+1}$ (and not $p_{u+1,N+1}$) and, hence, the corresponding diversity order associated with this term simplifies to $\zeta_{u,u+1}$.

In fact, for $v = u + 2$, the contribution of $\mathcal{I}_{n,i}$ to the outage probability in (14) can be written under the following form for large values of $\mathcal{P}_M$:

$$P_{\mathcal{I}_{n,i}} = \left[\prod_{j \in \mathcal{I}_{n,i}} p_{j,N+1} \prod_{j' \in \overline{\mathcal{I}}_{n,i}} p_{0,j'} \right] p(u) \prod_{\substack{u' \in \mathcal{I}_{n,i} \\ u' \neq u}} p(u') \quad (53)$$

where, from (22), $p(w) \triangleq p_{w \to w+1 \to \cdots \to w'-1}$ where, if $w = \mathcal{I}_{n,i,k}$, then $w' = \mathcal{I}_{n,i,k+1}$. For the case under consideration, $p(u) = p_{u \to u+1} \doteq p_{u,u+1} + p_{u+1,N+1}$ where $x \doteq y$ denotes that x is asymptotically equal to y. Consequently, (53) can be written as:

$$P_{\mathcal{I}_{n,i}} = \left[\prod_{j\in\mathcal{I}_{n,i}} p_{j,N+1} \prod_{j'\in\overline{\mathcal{I}}_{n,i}} p_{0,j'}\right] p_{u,u+1} \prod_{\substack{u'\in\mathcal{I}_{n,i}\\ u'\neq u}} p(u') +$$

$$\left[\prod_{j\in\mathcal{I}_{n,i}} p_{j,N+1} \prod_{j'\in\overline{\mathcal{I}}_{n,i}} p_{0,j'}\right] p_{u+1,N+1} \prod_{\substack{u'\in\mathcal{I}_{n,i}\\ u'\neq u}} p(u') \quad (54)$$

Consider the set $\mathcal{I}'_{n,i}=\mathcal{I}_{n,i}\cup\{u+1\}$. The contribution of this set to the outage probability is:

$$P_{\mathcal{I}'_{n,i}} = \left[\prod_{j\in\mathcal{I}'_{n,i}} p_{j,N+1} \prod_{j'\in\overline{\mathcal{I}'}_{n,i}} p_{0,j'}\right] p(u)p(u+1) \prod_{\substack{u'\in\mathcal{I}_{n,i}\\ u'\neq u}} p(u') \quad (55)$$

where, when calculated in the set $\mathcal{I}'_{n,i}$, $p(u) = p_{u\to u} = 1$ since $u+1 \in \mathcal{I}'_{n,i}$ and $p(u+1) = p_{u+1\to u+1} = 1$ since $u+2 = v \in \mathcal{I}_{n,i} \subset \mathcal{I}'_{n,i}$. Therefore:

$$P_{\mathcal{I}'_{n,i}} = \prod_{j\in\mathcal{I}'_{n,i}} p_{j,N+1} \prod_{j'\in\overline{\mathcal{I}'}_{n,i}} p_{0,j'} \prod_{\substack{u'\in\mathcal{I}_{n,i}\\ u'\neq u}} p(u') \quad (56)$$

$$= \frac{p_{u+1,N+1}}{p_{0,u+1}} \prod_{j\in\mathcal{I}_{n,i}} p_{j,N+1} \prod_{j'\in\overline{\mathcal{I}}_{n,i}} p_{0,j'} \prod_{\substack{u'\in\mathcal{I}_{n,i}\\ u'\neq u}} p(u') \quad (57)$$

Consequently, the second term in (54) can be written as $P_{\mathcal{I}'_{n,i}} p_{0,u+1}$ that is smaller than $P_{\mathcal{I}'_{n,i}}$. As a conclusion, when evaluating the diversity order, the probability in (54) can be approximated by the first term and, hence, $p(u) = p_{u\to u+1} \doteq p_{u,u+1}$.

Appendix B

Consider the third probability in (26). If $\mathcal{I}_{n,i,|\mathcal{I}_{n,i}|} = N$, then this probability is equal to $p_{N\to N} = 1$; otherwise, this probability will behave asymptotically as $p_{\mathcal{I}_{n,i,|\mathcal{I}_{n,i}|},\mathcal{I}_{n,i,|\mathcal{I}_{n,i}|}+1}$ based on the analysis presented in the case of IRC1. Similarly, the first probability in (26) will behave asymptotically as $p_{\mathcal{I}_{n,i,1},\mathcal{I}_{n,i,1}-1}$ for $\mathcal{I}_{n,i,1} \neq 1$; otherwise, this probability will be equal to 1.

Now, consider the probability of the form $p_{u\to(u+1\rightleftarrows\cdots\rightleftarrows v-1)\leftarrow v}$ where $u = \mathcal{I}_{n,i,l}$ and $v = \mathcal{I}_{n,i,l+1}$. The following cases arise. (i): $v = u+1$, in this case, the probability is equal to 1 following from the definition in (24). (ii): $v = u+2$; in this case the probability is $p_{u\to u+1\leftarrow v} = p_{u+1,N+1} + q_{u+1,N+1}p_{u,u+1}p_{v,u+1} \doteq p_{u+1,N+1} + p_{u,u+1}p_{v,u+1}$ where $x \doteq y$ means that x is asymptotically equal to y. Based on an analysis similar to that provided in Appendix A, it can be proven that the diversity order associated with this term is $\zeta_{u,u+1} + \zeta_{v,u+1}$ where in the outage probability P_{out}, the set $\mathcal{I}_{n,i}\cup\{u+1\}$ results in a probability that is $p_{0,u+1}$ times smaller than that obtained from the probability $p_{u+1,N+1}$ in $p_{u\to u+1\leftarrow v}$. In other words, P_{out} always contains a term that is smaller than $\left[\prod_{j\in\mathcal{I}_{n,i}} p_{j,N+1}\prod_{j'\in\overline{\mathcal{I}}_{n,i}} p_{0,j'}\right] p_{u+1,N+1}$. (iii): $v = u+3$; in this case, from (24), $p_{u\to(u+1\rightleftarrows v-1)\leftarrow v} = p_{u,u+1}p_{v\to v-1\to u+1} + q_{u,u+1}p_{u+1,N+1}p_{u+1\to v-1\leftarrow v} \doteq p_{u,u+1}p_{v,v-1} + p_{u+1,N+1}p_{u+1,v-1}p_{v,v-1} \doteq p_{u,u+1}p_{v,v-1}$. Therefore, by recursion, for $v \geq u+3$, $p_{u\to(u+1\rightleftarrows\cdots\rightleftarrows v-1)\leftarrow v} \doteq p_{u,u+1}p_{v,v-1}$ and the corresponding diversity order is $\zeta_{u,u+1} + \zeta_{v,v-1}$.

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