# Performance Analysis of Selective Relaying in Cooperative Free-Space Optical Systems

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***Abstract*—In this paper, we evaluate the performance of a Selective-Relaying (SR) protocol that is suitable for cooperative Free-Space Optical (FSO) communications with any number of relays. The SR strategy is based on transmitting all information symbols either along the direct link between the source and the destination or along one of the indirect links via the relays. We derive closed-form expressions of upper-bounds on the average error probability that can be achieved over Rayleigh and lognormal fading channels. We also provide an asymptotic analysis for quantifying the diversity gain and reduction in fading variance that can be realized by the cooperative scheme. The performance analysis shows the superiority of the SR scheme over the strategy that corresponds to activating all available links.**

***Index Terms*—Free-space optics, FSO, cooperation, relaying.**

## I. Introduction

Recently, there has been a growing interest in applying the cooperative diversity techniques in the context of Free-Space Optical (FSO) communications [1]–[14]. Cooperative diversity leverages the performance of FSO systems by mitigating the limiting effects of the turbulence-induced fading or scintillation. In this context, the limitations of multiple-aperture FSO systems that suffer form significant levels of channel correlation have motivated more research effort in the investigation of cooperative diversity as a promising alternative [15]. Despite the non-broadcast nature of FSO transmissions, high performance gains were reported in the literature. Moreover, the directivity of FSO links simplifies the design of cooperative networks since the transmissions from the different nodes do not interference with each other [1]–[14].

FSO relaying can be classified into two broad categories; namely, all-active and selective relaying. In all active-relaying, all relays decode (or amplify) the received symbols and simultaneously transmit the processed signals to the destination [1]–[11]. In selective-relaying, either direct transmissions occur or a single relay is selected among all relays for forwarding the information symbols to the destination depending on the channel state [12], [13]. In other words, instead of splitting the power for delivering the message to all relays, this power is combined and directed to the "best" relay in an attempt to enhance the fidelity of signal reconstruction at this relay. The criteria for selecting the best path were based on minimizing the conditional error probability for quantum-limited systems (in the absence of background noise) in [12] and on minimizing the outage probability for Gaussian noise in [13]. While all-active relaying is simple and does not require any kind of channel state information (CSI), selective-relaying achieves higher performance levels at the expense of acquiring the CSI. In this context, the solution in [14] can be perceived as a compromise between all-active and selective relaying where only the state of the source-relay link influences the role of the relay. On the other hand, for most of the existing cooperative FSO systems, the transmit power is evenly split among the active links [1]–[10]. In this context, optimized power allocation techniques are complicated and were proposed for multi-hop serial relaying systems in [11], for quantum-limited systems in [12] and for parallel-relaying systems corrupted by Gaussian noise in [13] where the solution comprised unfavorable numerical methods. In [6], instead of allocating the power for given relay locations, the positions of the relays were optimized with even power splitting.

In this work, we adopt the lognormal and Rayleigh models for the path gain distributions. The lognormal fading model is justified both by analysis in light turbulence and by empirical studies and has been considerably used in the analysis of cooperative FSO systems [1]–[6], [11], [12], [14]. Note that a lognormal distribution of the path gain $a$ results also in a lognormal distribution of the irradiance $I = a^2$. On the other hand, the Rayleigh model holds in the limit of strong turbulence [15], [16]. If the path gain follows the Rayleigh distribution, then the irradiance will follow the negative exponential distribution that has been extensively studied in the context of FSO communications [17], [18]. In this context, it is worth noting that a gamma-gamma irradiance will tend to a negative exponential irradiance (i.e. Rayleigh path gain) in the limit of strong turbulence [17]. Also the diversity order over gamma-gamma channels approaches that over Rayleigh channels as the link distance increases (strong turbulence) [9]. In our analysis, we have considered the two extremes, the lognormal and Rayleigh distributions, as an attempt to bound the range of performances observed in practice.

In this paper, we evaluate the error performance of selective-relaying based on a criterion that minimizes the conditional bit-error-probability (BEP). We first determine the distribution of the strongest path. At a second time, the average BEP was bounded by closed-form expressions that are useful in providing clear and intuitive insights on the performance of SR cooperative systems. Finally, based on an asymptotic analysis, we prove that a full diversity order of $N_r + 1$ can be achieved over Rayleigh channels. A more indicative parameter that

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quantifies the equivalent log-amplitude variance (or scintillation index) was proposed and derived in the case of lognormal fading. Unlike [12], the proposed analysis holds in the absence or presence of background noise. Moreover, while [12] targeted the power allocation problem where the performance was evaluated numerically, this paper targets the analytical evaluation of the average BEP. Finally, while [13] targeted the outage analysis of selective-relaying over gamma-gamma channels, our work investigates the BEP performance over lognormal and Rayleigh fading channels where the obtained analytical results are more conclusive in terms of providing comprehensive asymptotic analysis and tractable bounds as well as evaluating the impact of noise, the achievable diversity orders and the reduction in the scintillation index.

## II. System Model

Consider Binary Pulse Position Modulation (BPPM) where the symbol duration is divided into two slots with the optical pulse transmitted in only one of these slots. This modulation scheme is particulary popular with noncoherent FSO communications with intensity modulation and direct detection (IM/DD) because of its simplicity [1]–[6]. In this context, the receiver counts the number of photoelectrons in the two slots and decides in favor of the slot having the largest count. The average number of photoelectrons generated by the incident light signal (resp. background radiation and dark currents) in a PPM slot is denoted by $\lambda_s$ (resp. $\lambda_b$) where [15]:

$$\lambda_s = \eta \frac{P_r T_s/2}{hf} \triangleq \eta \frac{E_s}{hf} \quad ; \quad \lambda_b = \eta \frac{P_b T_s/2}{hf} \tag{1}$$

where $\eta$ is the detector's quantum efficiency assumed to be equal to 0.5, $h$ is Planck's constant and $f$ is the optical center frequency taken to be $1.94 \times 10^{14}$ Hz (corresponding to a wavelength of 1550 nm). $T_s$ stands for the symbol duration, $P_r$ for the incident optical power and $P_b$ for the power of background noise. Finally, $E_s = P_r T_s/2$ corresponds to the received optical energy per PPM slot along the direct link.

Consider the case where $N_r$ relays, denoted by R$_1$, ..., R$_{N_r}$, are present in the vicinity of a source node (S) and a destination node (D). Denote by $a_0$, $a_{1,1}, \dots, a_{1,N_r}$ and $a_{2,1}, \dots, a_{2,N_r}$ the path gains of the links S-D, S-R$_1, \dots,$ S-R$_{N_r}$ and R$_1$-D, ..., R$_{N_r}$-D, respectively. For Rayleigh fading, the probability density function (pdf) of the path gain $a$ is:

$$f_{\text{Ray}}(a) = 2ae^{-a^2} \quad ; \quad a \geq 0 \tag{2}$$

which results in a unity mean path intensity: $\text{E}[A^2] = 1$.

For lognormal fading, the pdf of $a$ is ($a \geq 0$):

$$f_{\text{LN}}(a; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma a} \exp\left( -\frac{(\ln(a) - \mu)^2}{2\sigma^2} \right) \tag{3}$$

where the parameters $\mu$ and $\sigma$ satisfy the relation $\mu = -\sigma^2$ so that the mean path intensity is unity. The log-amplitude variance of fading along a link of distance $d$ is given by [1]:

$$\sigma^2(d) \triangleq 0.124 k^{7/6} C_n^2 d^{11/6} \tag{4}$$

where $k$ is the wave number and $C_n^2$ denotes the refractive index structure constant [1].

Denote by $d_0$, $d_{1,n}$ and $d_{2,n}$ the lengths of the links S-D, S-R$_n$ and R$_n$-D, respectively. From (3), the pdf's of the path gains $a_0$ and $a_{m,n}$ can be written as $f_{\text{LN}}(a; -\sigma_0^2, \sigma_0)$ and $f_{\text{LN}}(a; -\sigma_{m,n}^2, \sigma_{m,n})$, respectively, where from (4) $\sigma_0 \triangleq \sigma(d_0)$ and $\sigma_{m,n} \triangleq \sigma(d_{m,n})$ for $m = 1, 2$ and $n = 1, \dots, N_r$.

Consider first the case where the direct link S-D is preferred over the $N_r$ indirect links. The decision at D will be based on the vector $\mathbf{Z_0} = [Z_{0,1}, Z_{0,2}]$ where $Z_{0,q}$ corresponds to the number of photoelectrons detected in the $q$-th PPM slot via the link S-D. For the PPM symbol $s$, a light signal is transmitted in slot $s$ and no signal is transmitted in slot $\bar{s}$ where: $\bar{s} = 2$ if $s = 1$ and $\bar{s} = 1$ if $s = 2$. In this case, the decision variables $Z_{0,s}$ and $Z_{0,\bar{s}}$ can be modeled as Poisson random variables (r.v.s) whose parameters are given by [15]:

$$\text{E}[Z_{0,s}] = a_0^2 \lambda_s + \lambda_b \triangleq k_0 + \lambda_b \quad ; \quad \text{E}[Z_{0,\bar{s}}] = \lambda_b \tag{5}$$

Now consider the case where the information is relayed by the $n$-th relay R$_n$. We denote the decision vector observed at this relay by $\mathbf{Z_n} = [Z_{n,1}, Z_{n,2}]$. In this case, $Z_{n,s}$ and $Z_{n,\bar{s}}$ are Poisson r.v.s with parameters (for $n \in \{1, \dots, N_r\}$):

$$\text{E}[Z_{n,s}] = \frac{1}{2}\beta_{1,n} a_{1,n}^2 \lambda_s + \lambda_b \triangleq \frac{1}{2} k_{1,n} + \lambda_b \quad ; \quad \text{E}[Z_{n,\bar{s}}] = \lambda_b \tag{6}$$

where $\beta_{1,n} = \left(\frac{d_0}{d_{1,n}}\right)^2 e^{-\sigma(d_{1,n} - d_0)}$ is a gain factor associated with the link S-R$_n$ where $\sigma$ is the attenuation coefficient [1].

Relay R$_n$ decides in favor of $\tilde{s} = \arg\max_{q=1,2}\{Z_{n,q}\}$ and forwards this symbol to D via the link R$_n$-D. The corresponding decision vector at D will be denoted by $\mathbf{Y_n} = [Y_{n,1}, Y_{n,2}]$ where the parameters of the Poisson r.v.s $Y_{n,\tilde{s}}$ and $Y_{n,\bar{\tilde{s}}}$ are:

$$\text{E}[Y_{n,\tilde{s}}] = \frac{1}{2}\beta_{2,n} a_{2,n}^2 \lambda_s + \lambda_b \triangleq \frac{1}{2} k_{2,n} + \lambda_b \quad ; \quad \text{E}\left[Y_{n,\bar{\tilde{s}}}\right] = \lambda_b \tag{7}$$

where $\beta_{2,n} = \left(\frac{d_0}{d_{2,n}}\right)^2 e^{-\sigma(d_{2,n} - d_0)}$ stands for the gain factor of link R$_n$-D. Finally, D decides in favor of the symbol $\hat{s} = \arg\max_{q=1,2}\{Y_{n,q}\}$. Note that the multiplying factor of $1/2$ was introduced in (6) and (7) since the power is evenly split among the two hops S-R$_n$ and R$_n$-D. This approach of evenly distributing the power among the active links is simple, robust, avoids estimating $\lambda_b$ and has been used widely with cooperative FSO systems [1]–[10].

## III. Conditional Error Probability

In this section, we derive the BEP conditioned on the channel state vector $A \triangleq [k_0, k_{1,1}, \dots, k_{1,N_r}, k_{2,1}, \dots, k_{2,N_r}]$. Note that the different components of $A$ are independent following from the independence between the path gains.

### A. Absence of background radiation

Consider first the direct link S-D. For $\lambda_b = 0$, $Z_{0,\bar{s}} = 0$ in (5). In this case, $Z_{0,s} > 0$ will ensure a correct detection while the random tie breaking when $Z_{0,s} = 0$ will result in an error with probability $1/2$. Consequently, the conditional BEP along the direct link S-D can be determined from:

$$P_{e|A}^{(0)} = \frac{1}{2}\text{Pr}(Z_{0,s} = 0) = \frac{1}{2} e^{-k_0} \tag{8}$$

Consider now the indirect link S-R$_n$-D. For $\lambda_b = 0$, $Y_{n,\bar{s}} = 0$ in (7). If $Y_{n,\tilde{s}} = 0$, then D breaks the tie randomly resulting in an erroneous decision with probability $1/2$. If $Y_{n,\tilde{s}} > 0$, then D decides in favor $\hat{s} = \tilde{s}$ resulting in a correct decision if $Z_{n,s} > 0$ (since in this case $\tilde{s} = s$) and in an erroneous decision with probability $1/2$ if $Z_{n,s} = 0$ (random tie breaking at R$_n$). Consequently, the conditional BEP along the $n$-th indirect link S-R$_n$-D can be written as:

$$P_{e|A}^{(n)} = \Pr\left(Y_{n,\tilde{s}} = 0\right) \times \frac{1}{2} + \Pr\left(Y_{n,\tilde{s}} > 0\right)\left[\Pr\left(Z_{n,s} > 0\right) \times 0 + \Pr\left(Z_{n,s} = 0\right) \times \frac{1}{2}\right] \quad (9)$$

Consequently, from (6) and (7):

$$P_{e|A}^{(n)} = \frac{1}{2}\left(e^{-\frac{1}{2}k_{1,n}} + e^{-\frac{1}{2}k_{2,n}} - e^{-\frac{1}{2}(k_{1,n}+k_{2,n})}\right) \quad (10)$$

In order to have an expression that is similar to (8) and for the sake of simplifying the calculations in the subsequent sections, (10) will be approximated as follows:

$$P_{e|A}^{(n)} \approx \frac{1}{2}\left(e^{-\frac{1}{2}k_{1,n}} + e^{-\frac{1}{2}k_{2,n}}\right) \approx \frac{1}{2}e^{-\left(\frac{1}{2}\min\{k_{1,n},k_{2,n}\}\right)} \quad (11)$$

where the above approximations (which actually correspond to upper-bounds) are tight following from the rapid exponential decay of the terms $e^{-\frac{1}{2}k_{1,n}}$ and $e^{-\frac{1}{2}k_{2,n}}$. This is especially true for large values of $\lambda_s$ where cooperation is the most useful.

For a given channel realization, the best link among the $N_r + 1$ links S-D and $\{\text{S-R}_n\text{-D}\}_{n=1}^{N_r}$ is the one that achieves the smallest conditional BEP. In this case, from (8) and (11), the conditional BEP of the SR scheme can be written as:

$$P_{e|A} = \min_{n=0,\dots,N_r}\{P_{e|A}^{(n)}\} \approx \frac{1}{2}e^{-\lambda} \quad (12)$$

where the random variable $\lambda$ is defined as:

$$\lambda = \max\left\{k_0, \left\{\frac{1}{2}\min\{k_{1,n}, k_{2,n}\}\right\}_{n=1}^{N_r}\right\} \quad (13)$$

### B. Presence of background radiation

At a first time, consider the direct link S-D. Ignoring the probability of correct decision when ties occur, the conditional BEP can be expressed as follows (for all values of $S \geq 0$):

$$P_{e|A}^{(0)} \approx \Pr\left(Z_{0,\bar{s}} \geq Z_{0,s}\right) \leq \Phi_{Z_{0,\bar{s}}}(S)\Phi_{Z_{0,s}}(-S) \quad (14)$$

where the Chernoff bound was applied. $\Phi_{Z_{0,\bar{s}}}(S) = e^{-\lambda_b(1-e^{-S})}$ and $\Phi_{Z_{0,s}}(S) = e^{-(k_0+\lambda_b)(1-e^{-S})}$ stand for the characteristic functions of the r.v.s $Z_{0,\bar{s}}$ and $Z_{0,s}$, respectively. Minimizing (14) with respect to $S$ results in:

$$P_{e|A}^{(0)} \approx e^{-\left(\sqrt{k_0+\lambda_b}-\sqrt{\lambda_b}\right)^2} \quad (15)$$

Consider now the indirect link S-R$_n$-D and denote by $P_{e,1|A}^{(n)}$ and $P_{e,2|A}^{(n)}$ the conditional probabilities of error along the hops S-R$_n$ and R$_n$-D, respectively. The conditional BEP along this link can be written as:

$$P_{e|A}^{(n)} = P_{e,1|A}^{(n)}[1-P_{e,2|A}^{(n)}] + P_{e,2|A}^{(n)}[1-P_{e,1|A}^{(n)}] \approx P_{e,1|A}^{(n)} + P_{e,2|A}^{(n)} \quad (16)$$

Performing an analysis similar to that performed in (14), and from (6) and (7), equation (16) can be written as:

$$P_{e|A}^{(n)} \approx e^{-\left(\sqrt{\frac{1}{2}k_{1,n}+\lambda_b}-\sqrt{\lambda_b}\right)^2} + e^{-\left(\sqrt{\frac{1}{2}k_{2,n}+\lambda_b}-\sqrt{\lambda_b}\right)^2} \quad (17)$$

$$\approx e^{-\left(\sqrt{\frac{1}{2}\min\{k_{1,n},k_{2,n}\}+\lambda_b}-\sqrt{\lambda_b}\right)^2} \quad (18)$$

Finally, the conditional BEP can be written as:

$$P_{e|A} = \min_{n=0,\dots,N_r}\{P_{e|A}^{(n)}\} \approx e^{-\left(\sqrt{\lambda+\lambda_b}-\sqrt{\lambda_b}\right)^2} \quad (19)$$

where $\lambda$ is given in (13). Note that (12) and (19) differ by a factor $1/2$ when $\lambda_b = 0$. This follows since the exact expression in (12) is derived based on the fact that ties are broken randomly resulting in an erroneous decision with probability $1/2$. On the other hand, in order to make use of the Chernoff bound, the upper-bound in (19) was derived assuming that ties will always result in errors.

## IV. Probability Density Function of $\lambda$

To be able to evaluate the average BEP in closed form, we need to evaluate the pdf $f_\lambda(.)$ of the r.v. $\lambda$ in (13). Following from the independence between the different path gains, the cumulative distribution function (cdf) of $\lambda$ can written as:

$$F_\lambda(t) = \Pr(\lambda \leq t) = \Pr(k_0 \leq t)\prod_{n=1}^{N_r}\Pr\left(\min\{k_{1,n}, k_{2,n}\} \leq 2t\right)$$

$$= \Pr(k_0 \leq t)\prod_{n=1}^{N_r}\left[1 - \prod_{m=1}^{2}\Pr\left(k_{m,n} > 2t\right)\right] \quad (20)$$

### A. Rayleigh Fading

Replacing $k_0$, $k_{1,n}$ and $k_{2,n}$ by their values from (5), (6) and (7), the last equation can be written as:

$$F_\lambda(t) = \Pr\left(a_0 \leq \sqrt{\frac{t}{\lambda_s}}\right) \times \prod_{n=1}^{N_r}\left[1-\Pr\left(a_{1,n} > \sqrt{\frac{2t}{\beta_{1,n}\lambda_s}}\right)\Pr\left(a_{2,n} > \sqrt{\frac{2t}{\beta_{2,n}\lambda_s}}\right)\right] \quad (21)$$

which in the case of Rayleigh fading results in:

$$F_\lambda(t) = \left(1 - e^{-\frac{t}{\lambda_s}}\right)\prod_{n=1}^{N_r}\left[1 - e^{-\frac{2t}{\beta_n\lambda_s}}\right] \quad (22)$$

where the equivalent gain factor $\beta_n$ of the link S-R$_n$-D is:

$$\frac{1}{\beta_n} \triangleq \frac{1}{\beta_{1,n}} + \frac{1}{\beta_{2,n}} \quad (23)$$

*1) 1 Relay:* Replacing $N_r = 1$ in (22), the pdf $f_\lambda(t) = \frac{\mathrm{d}F_\lambda(t)}{\mathrm{d}t}$ takes the following form:

$$f_\lambda(t) = \frac{1}{\lambda_s}\left[e^{-\frac{t}{\lambda_s}} + \frac{2}{\beta_1}e^{-\frac{2t}{\beta_1\lambda_s}} - \left(1+\frac{2}{\beta_1}\right)e^{-\left(1+\frac{2}{\beta_1}\right)\frac{t}{\lambda_s}}\right] \quad (24)$$

*2) 2 Relays:* For $N_r = 2$, differentiating (22) results in:

$$f_\lambda(t) = \frac{1}{\lambda_s}\left[e^{-\frac{t}{\lambda_s}} + \sum_{n=1}^{2}\left(\frac{2}{\beta_n}e^{-\frac{2t}{\beta_n\lambda_s}} - \left(1+\frac{2}{\beta_n}\right)e^{-\left(1+\frac{2}{\beta_n}\right)\frac{t}{\lambda_s}}\right) - \left(\frac{2}{\beta_1}+\frac{2}{\beta_2}\right)e^{-\left(\frac{2}{\beta_1}+\frac{2}{\beta_2}\right)\frac{t}{\lambda_s}} + \left(1+\frac{2}{\beta_1}+\frac{2}{\beta_2}\right)e^{-\left(1+\frac{2}{\beta_1}+\frac{2}{\beta_2}\right)\frac{t}{\lambda_s}}\right] \quad (25)$$

*3) $N_r$ Relays:* For $N_r > 2$, the calculations become intractable for any relay positions that result in any values of $\beta_1, \dots, \beta_{N_r}$. Consequently, we will consider the case of a "symmetrical" FSO network for which $\beta_1 = \dots = \beta_{N_r} \triangleq \beta$. For such networks, the distances $d_{1,n}$ and $d_{2,n}$ of R$_n$ from S and D satisfy the following relation:

$$(d_{1,n}, d_{2,n}) \in \{(D_1, D_2), (D_2, D_1)\} \quad ; \quad n = 1, \dots, N_r \quad (26)$$

for arbitrary values of the distances $D_1$ and $D_2$. A special case of the symmetrical network corresponds to the scenario where all relays are at the same distance from S and from D (i.e. $(d_{1,n}, d_{2,n}) = (D_1, D_2)$ for all values of $n$).

In this case, (22) simplifies to:

$$F_\lambda(t) = \left(1 - e^{-\frac{t}{\lambda_s}}\right)\left(1 - e^{-\frac{2t}{\beta\lambda_s}}\right)^{N_r} \quad (27)$$

resulting in:

$$f_\lambda(t) = \frac{1}{\lambda_s}\left[e^{-\frac{t}{\lambda_s}}\left(1 - e^{-\frac{2t}{\beta\lambda_s}}\right)^{N_r} + \frac{2N_r}{\beta} e^{-\frac{2t}{\beta\lambda_s}}\left(1 - e^{-\frac{t}{\lambda_s}}\right)\left(1 - e^{-\frac{2t}{\beta\lambda_s}}\right)^{N_r - 1}\right] \quad (28)$$

## B. Lognormal Fading

In the case of lognormal fading, $k_0$ in (5) can be modeled as a lognormal r.v. with parameters $[\ln(\lambda_s) - 2\sigma_0^2]$ and $2\sigma_0$. In the same way, from (6) and (7), $\frac{1}{2}k_{m,n}$ is a lognormal r.v. with parameters $[\ln(\frac{\beta_{m,n}}{2}\lambda_s) - 2\sigma_{m,n}^2]$ and $2\sigma_{m,n}$. In this case, (20) can be written as:

$$F_\lambda(t) = Q_0(t) \prod_{n=1}^{N_r}\left[1 - \prod_{m=1}^{2}(1 - Q_{m,n}(t))\right] \quad (29)$$

where:

$$Q_0(t) = Q\left(\frac{\ln(\lambda_s) - 2\sigma_0^2 - \ln(t)}{2\sigma_0}\right) \quad (30)$$

$$Q_{m,n}(t) = Q\left(\frac{\ln(\frac{\beta_{m,n}}{2}\lambda_s) - 2\sigma_{m,n}^2 - \ln(t)}{2\sigma_{m,n}}\right) \quad (31)$$

where $Q(x)$ is the Q-function: $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}\mathrm{d}t$.

Following from the rapid exponential decay of $Q(x)$ for large values of $x$, (29) can be approximated by:

$$F_\lambda(t) \approx Q_0(t) \prod_{n=1}^{N_r}[Q_{1,n}(t) + Q_{2,n}(t)] \quad (32)$$

*1) 1 Relay:* For $N_r = 1$, differentiating (32) results in:

$$f_\lambda(t) = \frac{1}{\sqrt{2\pi}2t}\left[\frac{1}{\sigma_0}f_0(t)\left[Q_{1,1}(t) + Q_{2,1}(t)\right] + Q_0(t)\left[\frac{1}{\sigma_{1,1}}f_{1,1}(t) + \frac{1}{\sigma_{2,1}}f_{2,1}(t)\right]\right] \quad (33)$$

where:

$$f_0(t) = \exp\left(-\frac{\left[\ln(t) - (\ln(\lambda_s) - 2\sigma_0^2)\right]^2}{8\sigma_0^2}\right) \quad (34)$$

$$f_{m,n}(t) = \exp\left(-\frac{\left[\ln(t) - (\ln(\frac{\beta_{m,n}}{2}\lambda_s) - 2\sigma_{m,n}^2)\right]^2}{8\sigma_{m,n}^2}\right) \quad (35)$$

Since $Q(x)$ can be upper-bounded as: $Q(x) \leq \frac{1}{2}e^{-\frac{x^2}{2}}$ for $x \geq 0$, then from (30)-(31) and (34)-(35):

$$Q_0(t) \leq \frac{1}{2}f_0(t) \quad ; \quad Q_{m,n}(t) \leq \frac{1}{2}f_{m,n}(t) \quad (36)$$

which hold since the arguments of the Q-functions in (30) and (31) are always positive since the bounds are derived for large values of $\lambda_s$. Consequently, (33) can be written as:

$$f_\lambda(t) \approx \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}2t}\left(\frac{1}{\sigma_0} + \frac{1}{\sigma_{1,1}}\right)f_0(t)f_{1,1}(t) + \frac{1}{\sqrt{2\pi}2t}\left(\frac{1}{\sigma_0} + \frac{1}{\sigma_{2,1}}\right)f_0(t)f_{2,1}(t)\right] \quad (37)$$

After manipulating the product $f_0(t)f_{m,n}(t)$, we can write this function under the following form that turns out to be very beneficial in evaluating the average BEP afterwards:

$$f_0(t)f_{m,n}(t) = \exp\left(-\frac{1}{8}g_{m,n}^{(eq)}\right)\exp\left(-\frac{(\ln(t) - \mu_{m,n}^{(eq)})^2}{8(\sigma_{m,n}^{(eq)})^2}\right) \quad (38)$$

where the equivalent log-amplitude variance is given by:

$$\frac{1}{(\sigma_{m,n}^{(eq)})^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_{m,n}^2} \quad (39)$$

and where the constant $g_{m,n}^{(eq)}$ does not depend on $\lambda_s$:

$$g_{m,n}^{(eq)} = \frac{1}{\sigma_0^2 + \sigma_{m,n}^2}\left[\ln(\beta_{m,n}/2) + 2(\sigma_0^2 - \sigma_{m,n}^2)\right]^2 \quad (40)$$

while $\mu_{m,n}^{(eq)}$ is the only term that depends on $\lambda_s$ according to:

$$\mu_{m,n}^{(eq)} = \ln(\lambda_s) + (\sigma_{m,n}^{(eq)})^2\left[\frac{1}{\sigma_{m,n}^2}\ln(\beta_{m,n}/2) - 4\right] \quad (41)$$

Finally, replacing (38) in (37) shows that the pdf of $\lambda$ can be written as the mixture of two lognormal distributions:

$$f_\lambda(t) = \frac{1}{2}\sum_{m=1}^{2}\left(\frac{1}{\sigma_0} + \frac{1}{\sigma_{m,1}}\right)\sigma_{m,1}^{(eq)}e^{-\frac{1}{8}g_{m,1}^{(eq)}}f_{\mathrm{LN}}(t; \mu_{m,1}^{(eq)}, 2\sigma_{m,1}^{(eq)}) \quad (42)$$

where $f_{\mathrm{LN}}(.)$ stands for the lognormal pdf defined in (3).

*2) 2 Relays:* For $N_r = 2$, differentiating (32) and applying the upper-bounds in (36) result in:

$$f_\lambda(t) \approx \frac{1}{4}\frac{1}{\sqrt{2\pi}2t}\sum_{m=1}^{2}\sum_{m'=1}^{2}\left(\frac{1}{\sigma_0} + \frac{1}{\sigma_{m,1}} + \frac{1}{\sigma_{m',2}}\right) \times f_0(t)f_{m,1}(t)f_{m',2}(t) \quad (43)$$

Through straightforward yet tedious calculations, it can be proven that the product $f_0(t)f_{m,n}(t)f_{m',n'}(t)$ can be written in a way analogous to (38) as:

$$f_0(t)f_{m,n}(t)f_{m',n'}(t) = \exp\left(-\frac{1}{8}g_{m,m',n,n'}^{(eq)}\right) \times \exp\left(-\frac{(\ln(t) - \mu_{m,m',n,n'}^{(eq)})^2}{8(\sigma_{m,m',n,n'}^{(eq)})^2}\right) \quad (44)$$

where, in a way similar to (39), the equivalent log-amplitude variance satisfies the following relation:

$$\frac{1}{(\sigma^{(eq)}_{m,m',n,n'})^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_{m,n}^2} + \frac{1}{\sigma_{m',n'}^2} \tag{45}$$

and:

$$g^{(eq)}_{m,m',n,n'} = \left(\frac{\ln\frac{\beta_{m,n}}{2}}{\sigma_{m,n}} - 2\sigma_{m,n}\right)^2 + \left(\frac{\ln\frac{\beta_{m',n'}}{2}}{\sigma_{m',n'}} - 2\sigma_{m',n'}\right)^2 + 4\sigma_0^2 - (\sigma^{(eq)}_{m,m',n,n'})^2 \left(\frac{\ln(\beta_{m,n}/2)}{\sigma^2_{m,n}} + \frac{\ln(\beta_{m',n'}/2)}{\sigma^2_{m',n'}} - 6\right)^2 \tag{46}$$

$$\mu^{(eq)}_{m,m',n,n'} = \ln(\lambda_s) + (\sigma^{(eq)}_{m,m',n,n'})^2 \left(\frac{\ln(\beta_{m,n}/2)}{\sigma^2_{m,n}} + \frac{\ln(\beta_{m',n'}/2)}{\sigma^2_{m',n'}} - 6\right) \tag{47}$$

Replacing (44) in (43) shows that $\lambda$ corresponds to the mixture of four lognormal distributions:

$$f_\lambda(t) = \frac{1}{4}\sum_{m=1}^{2}\sum_{m'=1}^{2}\left(\frac{1}{\sigma_0} + \frac{1}{\sigma_{m,1}} + \frac{1}{\sigma_{m',2}}\right)\sigma^{(eq)}_{m,m',1,2}\, e^{-\frac{1}{8}g^{(eq)}_{m,m',1,2}} f_{\mathrm{LN}}(t; \mu^{(eq)}_{m,m',1,2}, 2\sigma^{(eq)}_{m,m',1,2}) \tag{48}$$

*3) $N_r$ Relays:* For $N_r > 2$, an approach similar to that adopted in the cases of $N_r = 1$ and $N_r = 2$ becomes unfeasible given the increased complexity of the involved terms. Consequently, as in the case of Rayleigh fading, we consider the case of a symmetrical network where (26) holds. In this case, we denote $\beta_m = \left(\frac{d_0}{D_m}\right)^2 e^{-\sigma(D_m - d_0)}$ the gain associated with the distance $D_m$ in (26) for $m = 1, 2$. In the same way, from (4), we define $\sigma_m \triangleq \sigma(D_m)$. In this case, (32) can be written as:

$$F_\lambda(t) = Q_0(t)\left[Q_1(t) + Q_2(t)\right]^{N_r} \tag{49}$$

where $Q_0(t)$ is given in (30) while $Q_m(t)$ can be obtained from the function $Q_{m,n}(t)$ in (31) by dropping the subscript $n$. Differentiating (49) results in:

$$f_\lambda(t) = \frac{1}{\sqrt{2\pi}2t}\left[\frac{1}{\sigma_0} f_0(t)\left[Q_1(t) + Q_2(t)\right]^{N_r} + N_r Q_0(t)\left[\frac{1}{\sigma_1} f_1(t) + \frac{1}{\sigma_2} f_2(t)\right]\left[Q_1(t) + Q_2(t)\right]^{N_r - 1}\right] \tag{50}$$

Applying the bounds in (36) and invoking the binomial formula, (50) can be approximated by:

$$f_\lambda(t) \approx \frac{2^{-N_r}}{\sqrt{2\pi}2t}\left[\left(\frac{1}{\sigma_0} + \frac{N_r}{\sigma_1}\right) f_0 f_1^{N_r} + \left(\frac{1}{\sigma_0} + \frac{N_r}{\sigma_2}\right) f_0 f_2^{N_r} + \sum_{i=1}^{N_r-1}\left[\frac{1}{\sigma_0}\binom{N_r}{i} + \frac{N_r}{\sigma_1}\binom{N_r-1}{i-1} + \frac{N_r}{\sigma_2}\binom{N_r-1}{i}\right] f_0 f_1^i f_2^{N_r - i}\right] \tag{51}$$

The function $f_0(t) f_m^{N_r}(t)$ (for $m = 1, 2$) can be written in a way analogous to (38) as:

$$f_0(t) f_m^{N_r}(t) = \exp\left(-\frac{1}{8} g_m^{(2,eq)}\right)\exp\left(-\frac{(\ln(t) - \mu_m^{(2,eq)})^2}{8(\sigma_m^{(2,eq)})^2}\right) \tag{52}$$

where $\sigma_m^{(2,eq)}$, $g_m^{(2,eq)}$ and $\mu_m^{(2,eq)}$ can be obtained from (39), (40) and (41), respectively, by replacing $\sigma_{m,n}$ with $\frac{\sigma_m}{\sqrt{N_r}}$ and by replacing $\beta_{m,n}$ with $\beta_m e^{-2(1-\frac{1}{N_r})\sigma_m^2}$.

In the same way, the function $f_0(t) f_1^i(t) f_2^{N_r - i}(t)$ can be written in a way analogous to (44) as:

$$f_0(t) f_1^i(t) f_2^{N_r - i}(t) = \exp\left(-\frac{1}{8} g_i^{(3,eq)}\right)\exp\left(-\frac{(\ln(t) - \mu_i^{(3,eq)})^2}{8(\sigma_i^{(3,eq)})^2}\right) \tag{53}$$

where $\sigma_i^{(3,eq)}$, $g_i^{(3,eq)}$ and $\mu_i^{(3,eq)}$ can be obtained from (45), (46) and (47), respectively, by replacing $\sigma_{m,n}$ with $\frac{\sigma_1}{\sqrt{i}}$, $\sigma_{m',n'}$ with $\frac{\sigma_2}{\sqrt{N_r - i}}$, $\beta_{m,n}$ with $\beta_1 e^{-2(1-\frac{1}{i})\sigma_1^2}$ and $\beta_{m',n'}$ with $\beta_2 e^{-2(1-\frac{1}{N_r - i})\sigma_2^2}$.

Finally, replacing (52) and (53) in (51) results in (54) shown at the top of the next page showing that $f_\lambda(t)$ corresponds to the mixture of $N_r + 1$ lognormal distributions.

## V. Average Error Probability

From (12), the average BEP in the absence of background radiation can be written as:

$$P_e \approx \frac{1}{2}\int_0^{+\infty} e^{-t} f_\lambda(t)\,\mathrm{d}t \tag{55}$$

while in the presence of background noise, (19) implies that:

$$P_e \approx \int_0^{+\infty} e^{-(\sqrt{t+\lambda_b} - \sqrt{\lambda_b})^2} f_\lambda(t)\,\mathrm{d}t \tag{56}$$

### A. Rayleigh Fading

*1) BEP:* Following from the expressions of $f_\lambda(t)$ in (24), (25) and (28), it is useful to evaluate the following integral:

$$I(k) \triangleq \frac{1}{\lambda_s}\int_0^{+\infty} e^{-(\sqrt{t+\lambda_b} - \sqrt{\lambda_b})^2} e^{-\frac{k}{\lambda_s}t}\,\mathrm{d}t = \frac{1}{\lambda_s + k} \times \left[1 + 2e^{\frac{k^2\lambda_b}{\lambda_s(\lambda_s + k)}}\sqrt{\frac{\pi\lambda_b\lambda_s}{\lambda_s + k}}\, Q\left(\frac{k}{\lambda_s}\sqrt{\frac{2\lambda_b\lambda_s}{\lambda_s + k}}\right)\right] \tag{57}$$

where conventional integration techniques were applied. For $\lambda_b = 0$, the above integral simplifies to the following expression that is useful for evaluating the BEP in (55):

$$I(k) = \frac{1}{\lambda_s}\int_0^{+\infty} e^{-t} e^{-\frac{k}{\lambda_s}t}\,\mathrm{d}t = \frac{1}{\lambda_s + k} \tag{58}$$

Consequently, from (24), the average BEP with one relay can be written as:

$$P_e \approx I(1) + \frac{2}{\beta_1} I\left(\frac{2}{\beta_1}\right) - \left(1 + \frac{2}{\beta_1}\right) I\left(1 + \frac{2}{\beta_1}\right) \tag{59}$$

and, from (25), for $N_r = 2$:

$$P_e \approx I(1) + \sum_{n=1}^{2}\left[\frac{2}{\beta_n} I\left(\frac{2}{\beta_n}\right) - \left(1 + \frac{2}{\beta_n}\right) I\left(1 + \frac{2}{\beta_n}\right)\right] - \left(\frac{2}{\beta_1} + \frac{2}{\beta_2}\right) I\left(\frac{2}{\beta_1} + \frac{2}{\beta_2}\right) + \left(1 + \frac{2}{\beta_1} + \frac{2}{\beta_2}\right) I\left(1 + \frac{2}{\beta_1} + \frac{2}{\beta_2}\right) \tag{60}$$

$$f_\lambda(t) = 2^{-N_r}\left[\sum_{m=1}^{2}\left(\frac{1}{\sigma_0}+\frac{N_r}{\sigma_m}\right)\sigma_m^{(2,eq)} e^{-\frac{1}{8}g_m^{(2,eq)}} f_{\text{LN}}(t,\mu_m^{(2,eq)};2\sigma_m^{(2,eq)}) + \sum_{i=1}^{N_r-1}\left[\frac{1}{\sigma_0}\binom{N_r}{i}+\frac{N_r}{\sigma_1}\binom{N_r-1}{i-1}+\frac{N_r}{\sigma_2}\binom{N_r-1}{i}\right]\right]\sigma_i^{(3,eq)} e^{-\frac{1}{8}g_i^{(3,eq)}} f_{\text{LN}}(t,\mu_i^{(3,eq)};2\sigma_i^{(3,eq)}) \quad (54)$$

Finally, invoking the binomial formula in (28), the average BEP with $N_r$ symmetrical relays can be written as:

$$P_e \approx \sum_{i=0}^{N_r-1}\binom{N_r-1}{i}(-1)^i\left[I\left(1+\frac{2i}{\beta}\right) + \frac{2N_r}{\beta}I\left(\frac{2}{\beta}+\frac{2i}{\beta}\right)-\left(1+\frac{2N_r}{\beta}\right)I\left(1+\frac{2}{\beta}+\frac{2i}{\beta}\right)\right] \quad (61)$$

where in (59)-(61), $I(k)$ takes the value in (57) in the presence of background radiation. In the absence of background noise, $I(k)$ takes the value in (58) and the resulting BEP needs to be multiplied by $1/2$ (from (55)).

Note that, using the relation $Q(x) \leq \frac{1}{2}e^{-\frac{x^2}{2}}$, (57) can be bounded as $I(k) \leq \frac{1}{\lambda_s+k}\left[1+\sqrt{\frac{\pi\lambda_b\lambda_s}{\lambda_s+k}}\right]$ which tends to $\frac{1+\sqrt{\pi\lambda_b}}{\lambda_s+k}$ for large values of $\lambda_s$. Consequently, from (55) and (56), the presence of background radiation degrades the asymptotic error probability by a factor of $2[1+\sqrt{\pi\lambda_b}]$.

*2) Asymptotic Analysis and Diversity Order:* Equations (59)-(61) do not offer clear and intuitive insights on the performance of the cooperative system. In particular, the diversity order that can be achieved by the proposed scheme is not readily provided by these equations. Consequently, in what follows, we will proceed with an asymptotic analysis that holds for large values of $\lambda_s$.

For $\lambda_s \gg 1$, approximating $e^{-x}$ by $1-x$ (for small values of $x$) in (22) results in the following expression of the cdf:

$$F_\lambda(t) = \frac{2^{N_r}}{\lambda_s^{N_r+1}\prod_{n=1}^{N_r}\beta_n}t^{N_r+1} \triangleq \frac{2^{N_r}}{\beta_{eq}^{N_r}\lambda_s^{N_r+1}}t^{N_r+1} \quad (62)$$

where $\beta_{eq}$ stands for the equivalent gain factor of the network.

Differentiating (62) and replacing in (55) results in:

$$P_e \approx \frac{2^{N_r-1}(N_r+1)!}{\beta_{eq}^{N_r}\lambda_s^{N_r+1}} \quad (63)$$

which scales asymptotically as $\lambda_s^{-(N_r+1)}$ showing that the diversity order of the proposed scheme is equal to $N_r+1$ in the absence of background radiation.

In the presence of background radiation, differentiating (62) and replacing in (56) results in:

$$P_e \approx \frac{2^{N_r}(N_r+1)\gamma}{\beta_{eq}^{N_r}\lambda_s^{N_r+1}} \quad (64)$$

where ($\Gamma(.)$ stands for the gamma function):

$$\gamma = \sum_{i=0}^{N_r}\binom{N_r}{i}2^i\lambda_b^{\frac{i}{2}}\left[\Gamma\left(N_r-\frac{i}{2}+1\right)+\lambda_b^{\frac{1}{2}}\Gamma\left(N_r-\frac{i}{2}+\frac{1}{2}\right)\right] \quad (65)$$

As a conclusion, the SR cooperation scheme achieves a full diversity order of $N_r+1$ (which corresponds to the total number of paths between S and D) over Rayleigh fading channels whether in the absence or presence of background noise. Finally, note that the findings in this section hold for any relay positions (and not necessarily for symmetrical networks).

### B. Lognormal Fading

*1) BEP:* In the absence of background radiation, expressing $f_\lambda(t)$ as the mixture of several lognormal pdf's as in (42), (48) and (54) turns out to be very useful for evaluating the integral in (55). For $N_r = 1$ in the absence of background radiation, replacing (42) in (55) and integrating results in:

$$P_e \approx \frac{1}{4}\sum_{m=1}^{2}\left(\frac{1}{\sigma_0}+\frac{1}{\sigma_{m,1}}\right)\sigma_{m,1}^{(eq)}e^{-\frac{1}{8}g_{m,1}^{(eq)}}\text{Fr}(\lambda_s G_{m,1}^{(eq)},0;\sigma_{m,1}^{(eq)}) \quad (66)$$

where $\text{Fr}(a,0;b)$ is the lognormal density frustration function $\text{Fr}(a,0;b) = \int_0^{+\infty}\frac{1}{\sqrt{2\pi}bx}\exp\left(-ax^2\right)\exp\left[-\frac{(\ln(x)+b^2)^2}{2b^2}\right]\text{d}x$ [19]. $\sigma_{m,1}^{(eq)}$ and $g_{m,1}^{(eq)}$ are given in (39) and (40) while the gain term $G_{m,1}^{(eq)}$ is given by:

$$G_{m,1}^{(eq)} = e^{-2(\sigma_{m,1}^{(eq)})^2}\left(\beta_{m,1}/2\right)^{\left(\sigma_{m,1}^{(eq)}/\sigma_{m,1}\right)^2} \quad (67)$$

For $N_r = 2$, replacing (48) in (55) and integrating:

$$P_e \approx \frac{1}{8}\sum_{m=1}^{2}\sum_{m'=1}^{2}\left(\frac{1}{\sigma_0}+\frac{1}{\sigma_{m,1}}+\frac{1}{\sigma_{m',2}}\right)\sigma_{m,m',1,2}^{(eq)} e^{-\frac{1}{8}g_{m,m',1,2}^{(eq)}}\text{Fr}(\lambda_s G_{m,m',1,2}^{(eq)},0;\sigma_{m,m',1,2}^{(eq)}) \quad (68)$$

where $\sigma_{m,m',1,2}^{(eq)}$ and $g_{m,m',1,2}^{(eq)}$ are given in (45) and (46) while $G_{m,m',1,2}^{(eq)}$ takes the following from:

$$G_{m,m',1,2}^{(eq)} = e^{-4(\sigma_{m,m',1,2}^{(eq)})^2}\left[(\beta_{m,1}/2)^{\left(\sigma_{m,m',1,2}^{(eq)}/\sigma_{m,1}\right)^2}\times(\beta_{m',2}/2)^{\left(\sigma_{m,m',1,2}^{(eq)}/\sigma_{m',2}\right)^2}\right] \quad (69)$$

For $N_r$ relays, the average BEP in the absence of background noise can be obtained from multiplying (54) by $1/2$ and by replacing the functions $f_{\text{LN}}(t;\mu_m^{(2,eq)},2\sigma_m^{(2,eq)})$ and $f_{\text{LN}}(t;\mu_i^{(3,eq)},2\sigma_i^{(3,eq)})$ with $\text{Fr}(\lambda_s G_m^{(2,eq)},0;\sigma_m^{(2,eq)})$ and $\text{Fr}(\lambda_s G_i^{(3,eq)},0;\sigma_i^{(3,eq)})$, respectively. $G_m^{(2,eq)}$ can be obtained from (67) by replacing $\sigma_{m,1}$ with $\frac{\sigma_m}{\sqrt{N_r}}$ and by replacing $\beta_{m,1}$ with $\beta_m e^{-2(1-\frac{1}{N_r})\sigma_m^2}$. The gain $G_i^{(3,eq)}$ can be obtained from (69) by replacing $\sigma_{m,1}$ with $\frac{\sigma_1}{\sqrt{i}}$, $\sigma_{m',2}$ with $\frac{\sigma_2}{\sqrt{N_r-i}}$, $\beta_{m,1}$ with $\beta_1 e^{-2(1-\frac{1}{i})\sigma_1^2}$ and $\beta_{m',2}$ with $\beta_2 e^{-2(1-\frac{1}{N_r-i})\sigma_2^2}$.

In the presence of background noise, the integral in (56) does not admit a closed-form solution when $f_\lambda(t)$ comprises lognormal pdf's and hence this integral was solved numerically for the pdf's obtained in (42), (48) and (54).

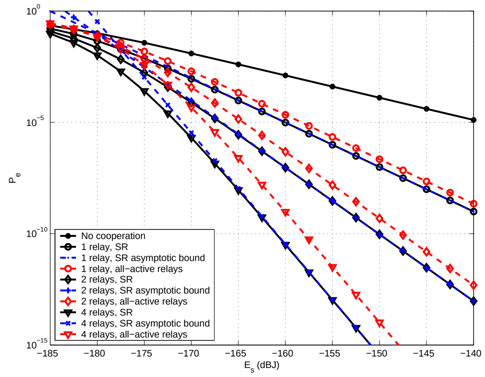


Fig. 1. Performance over Rayleigh channels for $P_b = 0$.

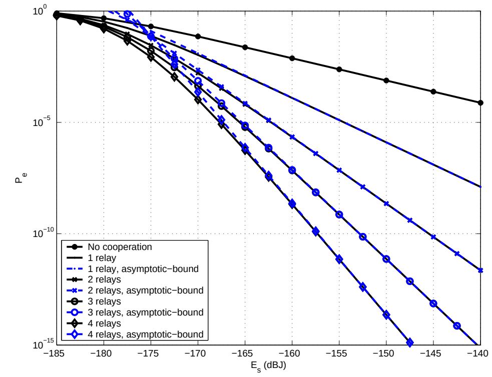


Fig. 2. Performance over Rayleigh channels for $P_b T_s/2 = -185$ dBJ.

*2) Asymptotic Analysis and Fading Reduction:* The frustration function can be bounded as $\text{Fr}(a, 0; b) \leq \frac{n!}{a^n} \exp\left(2(n^2+n)b^2\right)$ for $n \in \mathbb{N}$ [20]. Consequently, the error probabilities derived in the previous subsection and that comprise terms of the form $\text{Fr}(\lambda_s G, 0; \sigma)$ will scale asymptotically as $\lambda_s^{-n}$. Since this holds for all values of $n$, then the $P_e(\lambda_s)$ curves are infinitely steep for large values of $\lambda_s$ implying that the diversity order is not an accurate measure for the fading mitigation capabilities over lognormal fading channels. This behavior clearly distinguishes this type of channels from the Rayleigh channels where $P_e \to \lambda_s^{-d}$ where $d$ is the diversity order (which was proven to be $N_r + 1$ in subsection V.A).

Eq. (66) shows that the behavior of the cooperative system can be perceived as the composite behavior over two channels whose log-amplitude standard deviations (std's) are given by $\sigma_{1,1}^{(eq)}$ and $\sigma_{2,1}^{(eq)}$. Since the error rate will be dominated by the worst of the two channels (the one with higher std), then the equivalent log-amplitude std with one relay can be written as:

$$\sigma_{\text{eq}} = \max_{m=1,2}\{\sigma_{m,1}^{(eq)}\} = \left(\frac{1}{\sigma_0^2} + \frac{1}{(\max\{\sigma_{1,1}, \sigma_{2,1}\})^2}\right)^{-\frac{1}{2}} \quad (70)$$

In other words, the std of the logarithm of the path gain decreases from $\sigma_0$ for non-cooperative systems to the value given in (70) for a 1-relay system. In this context, (66) can be further approximated by:

$$P_e \approx \frac{1}{4}\left(\frac{1}{\sigma_0} + \frac{1}{\sigma_{\tilde{m},1}}\right) \sigma_{\tilde{m},1}^{(eq)} e^{-\frac{1}{8} g_{\tilde{m},1}^{(eq)}} \text{Fr}(\lambda_s G_{\tilde{m},1}^{(eq)}, 0; \sigma_{\tilde{m},1}^{(eq)}) \quad (71)$$

where $\tilde{m} = \arg\max_{m=1,2}(\sigma_{m,1})$ from (39) and where $\sigma_{\tilde{m},1}^{(eq)} = \sigma_{\text{eq}}$ in (70). In the same way, (68) can be written as:

$$P_e \approx \frac{1}{8}\left(\frac{1}{\sigma_0} + \frac{1}{\sigma_{\tilde{m},1}} + \frac{1}{\sigma_{\tilde{m}',2}}\right) \sigma_{\tilde{m},\tilde{m}',1,2}^{(eq)} e^{-\frac{1}{8} g_{\tilde{m},\tilde{m}',1,2}^{(eq)}} \text{Fr}(\lambda_s G_{\tilde{m},\tilde{m}',1,2}^{(eq)}, 0; \sigma_{\text{eq}}) \quad (72)$$

where $\tilde{m}' = \arg\max_{m'=1,2}(\sigma_{m',2})$ and for $N_r = 2$:

$$\sigma_{\text{eq}} = \left(\frac{1}{\sigma_0^2} + \frac{1}{(\max\{\sigma_{1,1}, \sigma_{2,1}\})^2} + \frac{1}{(\max\{\sigma_{1,2}, \sigma_{2,2}\})^2}\right)^{-\frac{1}{2}} \quad (73)$$

thus reflecting an additional reduction compared to (70).

Finally, with $N_r$ symmetrical relays it can be proven that:

$$P_e \approx 2^{-N_r-1}\left(\frac{1}{\sigma_0} + \frac{N_r}{\sigma_{\tilde{m}}}\right) \sigma_{\tilde{m}}^{(2,eq)} e^{-\frac{1}{8} g_{\tilde{m}}^{(2,eq)}} \text{Fr}(\lambda_s G_{\tilde{m}}^{(2,eq)}, 0; \sigma_{\text{eq}}) \quad (74)$$

where $\tilde{m} = \arg\max_{m=1,2}(\sigma_m)$ and:

$$\sigma_{\text{eq}} = \left(\frac{1}{\sigma_0^2} + \frac{N_r}{(\max\{\sigma_1, \sigma_2\})^2}\right)^{-\frac{1}{2}} \quad (75)$$

## VI. Numerical Results

The refractive index structure constant and the attenuation constant are set to $C_n^2 = 1 \times 10^{-14}$ $\text{m}^{-2/3}$ and $\sigma = 0.43$ dB/km. In all scenarios, the distance between S and D is $d_0 = 3$ km. For 1-relay systems, we set $d_{1,1} = 1$ km and $d_{2,1} = 2.5$ km. For 2-relay systems, the position of the first relay are the same as before while for the second relay we set $d_{1,2} = 1.5$ km and $d_{2,2} = 2$ km. For symmetrical $N_r$-relay systems with $N_r > 2$, we set $D_1 = 1$ km and $D_2 = 2.5$ km in (26). In the presence of background radiation, we present results for the case $P_b T_s/2 = -185$ dBJ in (1).

Figures 1 and 2 show the performance over Rayleigh-fading channels in the absence and presence of background radiation, respectively. Note that in this case, approximations were made on the conditional BEP but not on the pdf $f_\lambda(t)$. In these figures, we compare the bounds in (59)-(61) with the asymptotic bounds provided in (63) and (64). Results show that the upper-bounds provided in (59)-(61) are extremely close to the numerical results for all practical values of $E_s$ exceeding -180 dBJ (and hence these results were not presented in the figures for clarity). The results also show that the asymptotic bounds given in (63)-(64) can accurately predict the achievable performance levels and the diversity orders for large values of $E_s$. Finally, Fig. 1 shows the superiority of the SR over the all-active relaying especially for large number of relays.

Figures 3 and 4 show the performance over lognormal-fading channels in the absence and presence of background radiation, respectively. Results show that the bounds in (66) and (71) overlap for $N_r = 1$. The same holds for (68) and (72) on one hand and for (54) and (74) on the other hand for $N_r = 2$ and $N_r > 2$, respectively. Unlike the Rayleigh fading case where the proposed bounds overlap with the exact BEPs (for large values of $E_s$), the results show a certain gap in the

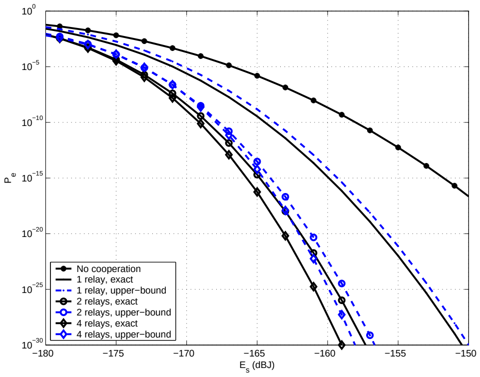


Fig. 3. Performance over lognormal channels for $P_b = 0$.

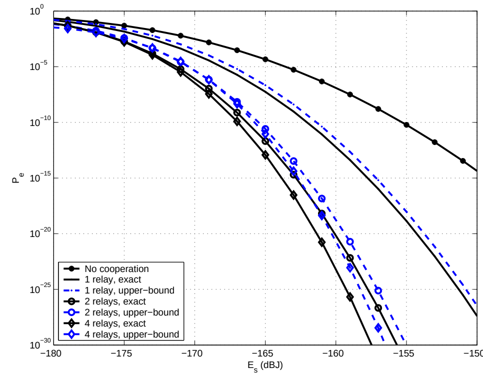


Fig. 4. Performance over lognormal channels for $P_b T_s/2 = -185$ dBJ.

case of lognormal fading. This follows since approximations were made on the pdf $f_\lambda(t)$ in order to make this function more appropriate for mathematical analysis. On the other hand, the proposed bounds are close enough, diversity-preserving and at a fixed offset from the exact results for large values of $E_s$. Note that while the log-amplitude std over the direct link is $\sigma_0 = 0.3878$, it drops to 0.2505, 0.1828 and 0.1511 in the cases of 1, 2 and 4 relays, respectively.

## VII. Conclusion

This paper analyzed the error performance of selective-relaying over Rayleigh and lognormal fading channels. SR can take advantage of the large coherence times of FSO channels for effectively acquiring the channel state information and advantageously activating the link that guarantees the smallest probability of error. The SR protocol is fully diverse over Rayleigh channels and results in significant reductions in the fading variance over distance-dependent lognormal channels.

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