Achieving Full Transmit Diversity for PPM Constellations with any Number of Antennas via Double Position and Symbol Permutations

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***Abstract*— We present a general technique for constructing minimal-delay rate-1 Space-Time (ST) block codes for Pulse Position Modulations (PPM) with an arbitrary number of transmit antennas. We show that the novel idea of time-domain constellation extension as well as introducing joint position and symbol permutations achieves a full transmit diversity order while maintaining unipolar transmissions. This renders the proposed scheme suitable for low-cost Time-Hopping Ultra-Wideband (TH-UWB) systems as well as Free-Space Optical (FSO) communications with direct detection.**

***Index Terms*— Space-Time (ST) block codes, PPM, Ultra-WideBand (UWB), Unipolar transmission.**

I. Introduction

The history of Space-Time (ST) coding is extensive. While initially considered with QAM , PAM and PSK [1], [2], there is a growing interest in applying the ST techniques with unipolar Pulse Position Modulation (PPM) [3]–[7]. This power-efficient modulation scheme found applications in Impulse-Radio Ultra-WideBand (IR-UWB) communications where it is complicated to control the amplitude and the phase of the very low duty-cycle sub-nanosecond modulated pulses [8]. PPM is also popular for Free-Space Optical (FSO) communications with direct detection where information can be only conveyed by the presence or absence of light pulses.

Two different approaches can be adopted for the construction of ST codes suitable for PPM constellations. The first approach consists of applying one of the numerous ST codes proposed in the literature for QAM, PAM or PSK [1], [2]. In this context, it has been shown that these codes remain fully diverse with PPM [9]. However, the disadvantage is that all of these codes introduce phase rotations or amplitude scaling in order to achieve a full transmit diversity order and, consequently, are not suitable for FSO and low-cost UWB transceivers. For example, while single-antenna PPM systems transmit unipolar pulses, applying the Alamouti code [1] with PPM necessitates the transmission of pulses having positive and negative polarities.

In order to overcome the above disadvantage, the second approach adopted in [4]–[7] consisted of constructing PPM-specific unipolar codes where information is conveyed only by the time delays of the transmitted pulses (having the same amplitudes). Consider the problem of ST block code design with M-ary PPM constellations and n transmit antennas. The main limitation of the sporadic codes in [4]–[7] is that they can not be applied for all values of M and n. For example, [4]–[6] are exclusive to binary PPM with 2, 4 and 2^k antennas respectively. On the other hand, the set of values of M for which the code in [7] is fully diverse diminishes as n increases. In particular, there are no unipolar ST codes in the literature that can be associated with the values of (M,n) for which M is small and n is large (refer to table-1 in [7]).

The key solution behind the design of ST codes for any number of transmit antennas was the introduction of constellation extensions [10], [11]. In particular, scaling the amplitude or rotating the phase of the QAM or PSK symbols [10] or introducing a rotation to the multi-dimensional QAM constellations [11] resulted in ST code designs that can be applied for a wide range of n. However, this approach results in high Peak-to-Average Power Ratios (PAPR). Moreover, such amplitude constellation extensions are prohibited if unipolar transmissions must be maintained.

In this paper, we propose a solution for the design of unipolar ST block codes that can be applied with M-PPM and n transmit antennas for all values of M and n. With respect to the existing unipolar PPM codes [4]–[7], the introduction of an additional constellation extension rendered such a solution possible. However, unlike the codes proposed in [10], [11] and all the codes that are based on lattice rotations [9], the proposed scheme is the first known code that is based exclusively on a time-domain (position) constellation extension. In particular, unipolar transmissions are maintained and a full transmit diversity order is achieved by the introduction of an additional modulation position. In other words, for conveying a M-ary PPM symbol, each symbol duration must contain $M+1$ time slots (or positions) rather than M slots as in [4]–[7] or with single-antenna systems. This time-domain constellation extension corresponds to an increase (by the value of one) of the dimensionality of the transmitted signal.

The disadvantage of the proposed solution is an increased receiver complexity since an additional correlator must be added after each receiver antenna to collect the energy in the $(M+1)$-th position (as will be explained later). However, the advantage is that the proposed codes are the first general constructions that guarantee full spatial diversity for an arbitrary number of transmit antennas. For example, we propose the first known codes that can be associated with binary PPM ($M=2$) for $n=3,5,6$ or 7 antennas. Even though for these values of (M,n) the solution that we propose necessitates $M+1=3$

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positions, this constitutes a main contribution since there are no codes that are fully diverse with 3 modulation positions for $n > 3$ [7].

Note that it is not surprising that the additional constellation extension will result in a certain disadvantage since it corresponds to a mathematical tool that introduces an additional degree of freedom that renders the ST code construction possible. Note also that a constellation extension can be introduced either in the amplitude domain or the time domain. Since amplitude extensions are prohibited with unipolar transmissions, the only possible remaining solution (whether in this paper or in all future work) resides in a time-domain extension. This is equivalent to an increased constellation dimensionality and, consequently, an increased receiver complexity since the basis on which the received signal must be projected will have additional components.

II. System Model and Code Construction

Consider the case of M-ary PPM with $P = n$ transmit antennas. In what follows, we propose a minimal-delay linear transmit diversity scheme that extends over $T = n$ symbol durations. For conveying the n information symbols $p_1, \dots, p_n \in \{1, \dots, M\}$, each symbol duration is divided into $M+1$ positions (rather than M positions for M-PPM). Based on this new partitioning, the position of the pulse transmitted from i-th transmit antenna during the j-th symbol duration will be determined from the (i,j)-th element of the $n \times n$ matrix $\mathcal{P}$ given by:

$$\mathcal{P}_{i,j} = \begin{cases} p_{\sigma^{i-1}(j)}, & j \geq i; \\ \pi^{-1}\left(p_{\sigma^{i-1}(j)}\right), & j < i. \end{cases} \quad (1)$$

where $\pi^k(.)$ and $\sigma^k(.)$ stand for the cyclic permutations of order k over the $(M+1)$ extended positions and n symbol durations respectively:

$$\pi^k(i) = (i - k - 1) \mod (M+1) + 1 \quad (2)$$

$$\sigma^k(i) = (i - k - 1) \mod n + 1 \quad (3)$$

Equation (1) shows that the proposed diversity scheme corresponds to introducing convenient permutations among the positions and symbol durations justifying the title of the paper. Even though $p_i \in \{1, \dots, M\}$ for $i = 1, \dots, n$, eq. (2) shows that $\pi^{-1}(p_i) \in \{1, \dots, M+1\}$. This corresponds to the time-domain constellation extension introduced by the proposed scheme. Note that eq. (1) is not the only possible solution; moreover, it is not the optimal solution. This equation corresponds simply to a specific solution that we propose and that is appealing since it can be applied with any number of transmit antennas and modulation positions.

Based on eq. (1), the signal transmitted from the i-th antenna can be written as ($i = 1, \dots, P$):

$$s_i(t) = \frac{1}{\sqrt{PN_f}} \sum_{j=1}^{n} \sum_{n_f=0}^{N_f-1} w\left(t - (j-1)N_f T_f - n_f T_f - (\mathcal{P}_{i,j} - 1)\delta\right) \quad (4)$$

where $w(t)$ is the pulse waveform of duration T_w. N_f pulses are used to convey each symbol. Each one of these pulses is emitted during one time frame of duration T_f. δ is the modulation delay and is chosen to verify $\delta = T_w$. From equations (2)-(4), it can be seen that the proposed code introduces no amplitude scaling, phase rotations or multi-pulse transmissions per symbol duration. In other words, only one unipolar pulse is transmitted per symbol duration which constitutes the strength of the code.

For M-PPM, Inter-Symbol-Interference (ISI) is eliminated by choosing $T_f \geq (M-1)\delta + T_c$ where T_c stands for the channel delay spread ($T_c \gg \delta$). Since the proposed encoding scheme adds a new modulation position, T_f must now verify the relation: $T_f \geq M\delta + T_c$. Since the order of magnitude of T_c is about 100 ns [12] and since $\delta = T_w$ is in the order of 1 ns for UWB transmissions, then we deduce that the introduced constellation extension is associated, practically, with no reduction in the data rate.

For a system equipped with Q receive antennas and a L-th order Rake, the linear dependence between the baseband inputs and outputs of the channel can be expressed as [9]:

$$Y = HC + N \quad (5)$$

where Y and N are $QLM' \times T$ matrices corresponding to the decision variables and the noise terms respectively with $M' \triangleq M+1$. Note that M' (rather than M) correlators are needed per receive antenna and Rake finger since the encoded pulses can occupy M' positions. H is the $QLM' \times PM'$ channel matrix whose $((q-1)LM' + (l-1)M' + m, (p-1)M' + m')$-th element corresponds to the impact of the signal transmitted during the m'-th position of the p-th antenna on the m-th correlator (corresponding to the m-th position) placed after the l-th Rake finger of the q-th receive antenna.

C is the $PM' \times T$ codeword with $P = T = n$. From equations (1)-(3), C can be written as:

$$C(S_1, \dots, S_n) = \left[(C^{(0)}(S_1, \dots, S_n))^T \ \cdots \ (C^{(n-1)}(S_1, \dots, S_n))^T \right]^T \quad (6)$$

$$C^{(i)}(S_1, \dots, S_n) = \begin{bmatrix} S_{n-i+1,\pi(1)} & \cdots & S_{n,\pi(1)} & S_{1,1} & \cdots & S_{n-i,1} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ S_{n-i+1,\pi(M')} & \cdots & S_{n,\pi(M')} & S_{1,M'} & \cdots & S_{n-i,M'} \end{bmatrix} \quad (7)$$

where $C^{(i)}$ is a $M' \times n$ matrix for $i = 0, \dots, n-1$. $S_1, \dots, S_n$ are M'-dimensional vectors that belong to the set:

$$\mathcal{C} = \{e_m \ ; \ m = 1, \dots, M\} \quad (8)$$

where e_m is the m-th column of the $M' \times M'$ identity matrix. In other words, $S_i = e_{p_i}$ for $i = 1, \dots, n$. The m-th component of S_i is denoted by $S_{i,m}$ for $m = 1, \dots, M'$.

Proposition 1: [main result] The proposed code permits to achieve a full transmit diversity order with M-PPM and n transmit antennas for all values of M and n.

Proof: Based on [2] and because of the linearity of the code, the ST code is fully diverse if the matrix $C(X_1, \dots, X_n)$ has full rank for $(X_1, \dots, X_n) \in \mathcal{X}^n \backslash \{(\mathbf{0}_{M'}, \dots, \mathbf{0}_{M'})\}$ where

$$C' = \begin{bmatrix} C_m^{(k)} \\ \vdots \\ C_m^{(n-1)} \\ C_{\pi(m)}^{(0)} \\ \vdots \\ C_{\pi(m)}^{(k-1)} \end{bmatrix} = \begin{bmatrix} X_{n',\pi(m)} & \cdots & \cdots & X_{n,\pi(m)} & X_{1,m} & \cdots & X_{n'-1,m} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ X_{2,\pi(m)} & \cdots & X_{n',\pi(m)} & X_{n'+1,\pi(m)} & \cdots & X_{n,\pi(m)} & X_{1,m} \\ X_{1,\pi(m)} & \cdots & \cdots & X_{n',\pi(m)} & X_{n'+1,\pi(m)} & \cdots & X_{n,\pi(m)} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ X_{n'+1,\pi^2(m)} & \cdots & X_{n,\pi^2(m)} & X_{1,\pi(m)} & \cdots & \cdots & X_{n',\pi(m)} \end{bmatrix} \tag{12}$$

$\mathbf{0}_{M'}$ is the M'-dimensional zero vector and $\mathcal{X}$ corresponds to the set of possible differences between the elements of $\mathcal{C}$ given in eq. (8): $\mathcal{X} = \{s - s' \ ; \ s, s' \in \mathcal{C}\}$. The diversity order is achieved because of this particular structure of $\mathcal{X}$.

$C(X_1, \dots, X_n)$ will be denoted by C for simplicity while $C_j^{(i)}$ stands for the j-th row of $C^{(i)}$. We first observe that det$(C^T C)$ verifies the following relation:

$$\begin{aligned} \det(C^T C) &= \sum_{i_1=1}^{nM'} \sum_{i_2=i_1+1}^{nM'} \cdots \sum_{i_n=i_{n-1}+1}^{nM'} \left(\det\left(\left[C_{i_1}^T \ \cdots \ C_{i_n}^T\right]^T\right)\right)^2 \\ &\geq \sum_{i_1=1}^{M'} \sum_{i_2=M'+1}^{2M'} \cdots \sum_{i_n=(n-1)M'+1}^{nM'} \left(\det\left(\left[C_{i_1}^T \ \cdots \ C_{i_n}^T\right]^T\right)\right)^2 \\ &= \sum_{i_1=1}^{M'} \sum_{i_2=1}^{M'} \cdots \sum_{i_n=1}^{M'} \left(\det\left(\left[(C_{i_1}^{(0)})^T \ \cdots \ (C_{i_n}^{(n-1)})^T\right]^T\right)\right)^2 \end{aligned} \tag{9}$$

The proof is based on the following propositions. In what follows, $m \in \{1, \dots, M'\}$.

Proposition 2: $X_{1,m} = \cdots = X_{n-1,m} = 0$ implies that det$(C^T C) \geq 1$ unless $X_{n,m} = 0$.

Proof: Construct the $n \times n$ matrix $\mathcal{C}'$ as:

$$C' = \left[\ \left(C_{\pi^{-1}(m)}^{(1)}\right)^T \ \cdots \ \left(C_{\pi^{-1}(m)}^{(n-1)}\right)^T \ \ \left(C_m^{(0)}\right)^T \ \right]^T \tag{10}$$

$$= \begin{bmatrix} X_{n,m} & X_{1,\pi^{-1}(m)} & \cdots & \cdots & X_{n-1,\pi^{-1}(m)} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ X_{2,m} & \cdots & \cdots & X_{n,m} & X_{1,\pi^{-1}(m)} \\ X_{1,m} & \cdots & \cdots & X_{n-1,m} & X_{n,m} \end{bmatrix} \tag{11}$$

Since $X_{1,m} = \cdots = X_{n-1,m} = 0$, C' is upper triangular. From eq. (9), this implies that: $\det\left(C^T C\right) \geq \left(\det\left(C'\right)\right)^2 = (X_{n,m})^{2n}$. Moreover, based on the structure of the set $\mathcal{X}$, $X_{n,m} \in \{0, \pm 1\}$ completing the proof of proposition 2.

Proposition 3: $X_{1,m} = \cdots = X_{n'-1,m} = X_{n'+1,\pi(m)} = \cdots = X_{n,\pi(m)} = 0$ implies that det$(C^T C) \geq 1$ unless $X_{n',\pi(m)} = 0$.

Proof: Let $k \triangleq \sigma^{-1}(1 - n')$, we observe that the matrix C' given in eq. (12) at the top of the page is a lower triangular matrix. Consequently, from eq. (9): $\det\left(C^T C\right) \geq \left(\det\left(C'\right)\right)^2 = (X_{n',\pi(m)})^{2n}$ completing the proof.

Now the proof of the main proposition will follow from proposition 2 and proposition 3. Since none of the n information symbols initially occupies the position $M' = M + 1$ (before applying the pulse permutation), then $X_{1,M'} = \cdots = X_{n,M'} = 0$.

Since $X_{1,M'} = \cdots = X_{n-1,M'} = 0$, applying proposition 3 with $n' = n$ and $m = M'$ results in det$(C^T C) \geq 1$ except when $X_{n,\pi(M')} = 0$. Now, $X_{1,M'} = \cdots = X_{n-2,M'} = 0$ and $X_{n,\pi(M')} = 0$ imply that $X_{n-1,\pi(M')} = \cdots = X_{1,\pi(M')} = 0$ from the recursive application of proposition 3 for $n' = n-1, \dots, 1$. In other words, det$(C^T C) \geq 1$ except when the M'-th and $\pi(M')$-th components of $X_1, \dots, X_n$ are all equal to zero. Repeating the same procedure with $\pi(M')$ rather than M' we conclude that det$(C^T C) \geq 1$ except when the $\pi(M')$-th and $\pi^2(M')$-th components of $X_1, \dots, X_n$ are all equal to zero. By recursion, we conclude that det$(C^T C) \geq 1$ except when the components $\{M', \pi^1(M'), \dots, \pi^M(M')\}$ of $X_1, \dots, X_n$ are equal to zero implying that $X_1 = \cdots = X_n = \mathbf{0}_{M'}$ since this last set is equal to $\{1, \dots, M\}$.

III. Simulations and results

Simulations are performed over the IEEE 802.15.3a channel model recommendation CM2 [12]. A Gaussian pulse with a duration of $T_w = 0.5$ ns is used. The modulation delay is chosen to verify $\delta = T_w = 0.5$ ns and we fix $T_f = 100$ ns in order to eliminate the ISI.

Fig. 1 and Fig. 2 compare the performance of single-antenna systems and $P \times Q$ ST-coded systems with L-fingers Rakes and M-PPM for various values of P, Q, L, M. The Symbol-Error-Rate (SER) plots show the high performance levels and the enhanced diversity order achieved by the proposed scheme.

To highlight the advantages of ST coding with UWB systems, Fig. 3 compares systems having the same overall diversity order that is equal to the product PQL (with $Q = 1$ in this case). Binary PPM is used and for a fair comparison, we plot the BER as a function of PL rather than L. Results show that exploiting the transmit diversity by increasing the number of transmit antennas can be more beneficial than enhancing the multi-path diversity by increasing the number of Rake fingers even though there is no increase in the energy capture. For example, a 1×1 system equipped with 60 fingers achieves a BER of 4×10^{-3} at a SNR of 20 dB. In this case, the 2×1 system with only 30 fingers achieves a better BER in the order of 10^{-4} while the 3×1 system with 20 fingers achieves a BER of about 3×10^{-5}.

IV. Conclusion

We introduced a new construction method for unipolar ST codes with any number of antennas and signal-set dimensionality. The novel idea of time-domain constellation extension renders such construction possible and opens the door to more sophisticated and more powerful codes.

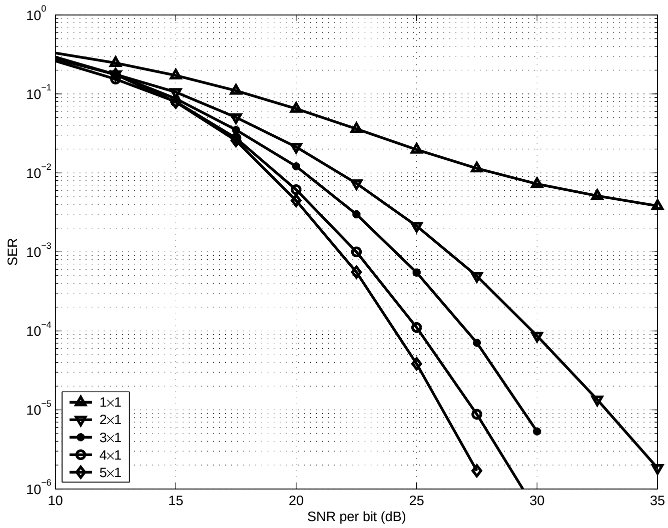

Fig. 1. Performance of the proposed code with P transmit antennas and 4-PPM for $P = 2, \dots, 5$. One receive antenna and a 1-finger Rake are used.

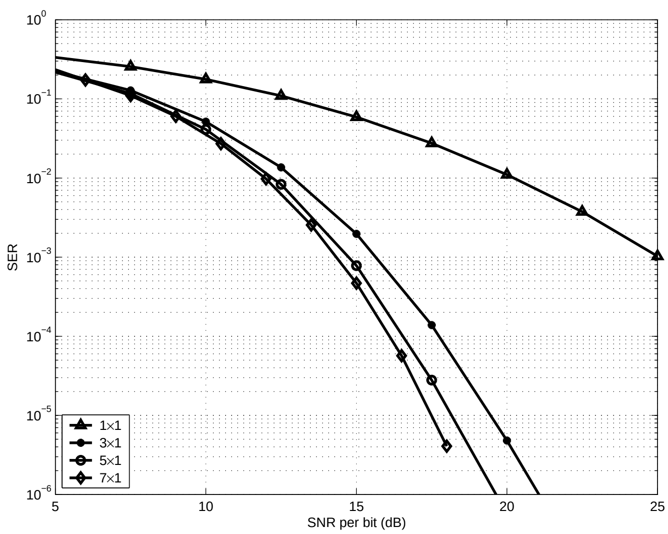

Fig. 2. Performance of the proposed code with P transmit antennas and 3-PPM for $P = 3, 5, 7$. One receive antenna and a 10-finger Rake are used.

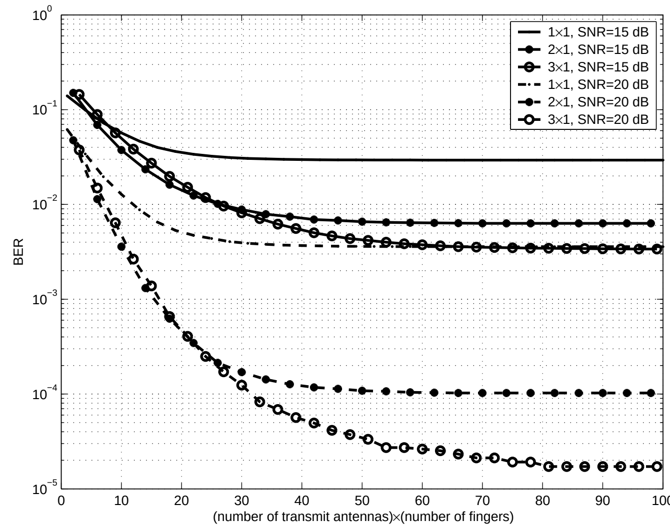

Fig. 3. Transmit diversity versus multi-path diversity with binary PPM and one receive antenna.

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