# Orthogonal Space-Time Block Codes for Binary Pulse Position Modulation

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***Abstract*— In this paper, we propose orthogonal Space-Time (ST) codes for binary Pulse Position Modulations (PPM). Unlike the well known orthogonal ST codes, the proposed schemes verify the additional constraint of achieving a full transmit diversity order without introducing any phase rotations. This renders the proposed codes suitable for Free-Space Optical (FSO) communications with direct detection and for Ultra-WideBand (UWB) communications. At the receiver side, optimal detection can be achieved with linear operations and the proposed codes can be also applied with On-Off Keying (OOK).**

***Index Terms*— Free-Space Optical (FSO) communication, Ultra-WideBand (UWB), Space-Time (ST) coding, PPM.**

## I. Introduction

Initially designed for Radio Frequency (RF) communications, Space-Time (ST) block codes are becoming more popular for Free-Space Optical (FSO) communications. Recent studies showed that ST coding can be a possible solution for solving the 'last mile' problem since spatial diversity can combat the atmospheric turbulence that degrades the performance of FSO links [1]–[3].

The literature of ST-RF coding is huge [4]–[6]. However, these codes are based on polarity inversions or amplitude amplifications and are, consequently, not adapted to FSO communications with direct detection using unipolar Pulse Position Modulations (PPM) or On-Off Keying (OOK). Consider for example the orthogonal codes [5]. The entries of the codewords are equal to $\pm s_i$ or $\pm s_i^*$ where $s_1, \dots, s_n$ are the information symbols and $n$ is the number of transmit antennas. While these codes are shape-preserving with QAM, they introduce a constellation extension when associated with PPM since $-s$ and $s$ can not be both PPM symbols simultaneously.

A first attempt in the FSO ST code design was made in [1] where the Alamouti code was tailored to binary PPM. It was shown in [7] that the extension of this scheme to $M$-ary PPM breaks down the orthogonality between the transmitted data streams. Shape-preserving ST codes for binary PPM with $2^k$ transmit antennas were proposed in [8] in the context of Time-Hopping Ultra-Wideband (TH-UWB) communications. However, this family of codes is not orthogonal.

On the other hand, despite the intensive research in designing more sophisticated families of ST block codes [6], the orthogonal ST codes [4], [5] remain appealing because of their simple decoding strategy. In this paper, we propose the construction of orthogonal ST codes that are shape-preserving with binary PPM. We show that for orthogonal constellations and in the absence of Inter-Position-Interference (IPI), maximum-likelihood detection of the proposed schemes can be achieved with linear operations. IPI breaks down the structure of the code necessitating the implementation of more sophisticated non-linear multi-dimensional 2-PPM decoders [8] at the receiver. The proposed schemes can be directly extended to OOK. The possibility of achieving a full transmit diversity order while transmitting unipolar pulses renders the proposed schemes appealing not only for FSO links but also for TH-UWB communications where it is difficult to control the phase (and the amplitude) of the very low duty cycle sub-nanosecond pulses. Following from this additional phase constraint that must be verified by both FSO and TH-UWB systems, we consider only the real orthogonal designs in what follows.

*Notations*: $0_n$ and $1_n$ correspond to the $n$-dimensional vectors whose components are equal to $0$ and $1$ respectively. vec$(X)$ stacks the columns of the matrix $X$ vertically. $\otimes$ stands for the Kronecker product. $I_M$ and $\mathbf{0}_M$ stand for the $M \times M$ identity matrix and the all-zero matrix respectively. $||X||^2$ stands for the Frobenius norm of the matrix $X$.

## II. System Model

Consider a FSO system where the transmitter and the receiver are equipped with $P$ laser sources and $Q$ photodetectors respectively. 2-PPM is a 2-dimensional constellation given by:

$$\mathcal{C} = \{[1 \quad 0]^T \ ; \ [0 \quad 1]^T\} \tag{1}$$

where the scalar 1 indicates the presence of a light waveform.

In the high signal-to-noise ratio (SNR) regime, the system performance is limited by the shot noise whose impact increases with the optical power of the signal incident on the receiver. Given that in this regime the Poisson photon arrivals can be approximated by Gaussian noise, then the linear dependence between the input and output of the Multiple-Input-Multiple-Output (MIMO) FSO channel can be expressed as:

$$X = HC + N \tag{2}$$

where $C$ is the $PM \times T$ codeword ($M = 2$) whose $((p-1)M + m, t)$-th entry corresponds to the amplitude of the pulse (if any) transmitted at the $m$-th position of the $p$-th source during the $t$-th symbol duration for $p = 1, \dots, P$, $m = 1, \dots, M$ and $t = 1, \dots, T$. The matrices $X$ and $N$ are $QLM \times T$ matrices corresponding to the decision variables and the noise terms respectively.

$H$ is the $QM \times PM$ channel matrix whose $(q, p)$-th $M \times M$ constituent sub-matrix is denoted by $H_{q,p}$ for $p = 1, \dots, P$

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and $q = 1, \ldots, Q$. The $(m, m')$-th element of $H_{q,p}$ corresponds to the contribution of the signal transmitted by the $p$-th source during the $m'$-th position that results when the waveform received at the $q$-th detector is projected onto the $m$-th basis vector for $m, m' = 1, \ldots, M$. In the absence of IPI, $H_{q,p} = h_{q,p} \otimes I_M$ where $h_{q,p}$ stands for the scintillation at the optical path between the $p$-th source and the $q$-th detector.

The relation given in eq. (2) also holds with TH-UWB systems. In this case, the highly frequency selective channels can be generated according to the IEEE 802.15.3a channel model recommendation [9] (interested readers are referred to [8] for more details on the system model of MIMO-TH-UWB systems). The only difference between FSO and TH-UWB is in the structure of the channel matrix $H$. Since the coding and decoding strategies are related to the modulation scheme (and not the channel matrix), the following results are valid for both FSO and TH-UWB.

## III. Code Construction

For $M$-PPM constellations with $n = P$ sources (and $M = 2$), we propose to construct the $nM \times n'$ codewords ($n' = T$) from the $n \times n'$ codewords based on the orthogonal design [5] by replacing the entries of these codewords that are equal to $-s_i$ by $\Omega s_i$ for $i = 1, \ldots, n'$. $\Omega$ is the $2 \times 2$ cyclic permutation matrix given by:

$$\Omega = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{3}$$

For example, for $n = 2, 4$, we propose the following structure for the minimal-delay $nM \times n$ codewords:

$$C(s_1, s_2) = \begin{bmatrix} s_1 & s_2 \\ \Omega s_2 & s_1 \end{bmatrix} \quad ;$$

$$C(s_1, s_2, s_3, s_4) = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ \Omega s_2 & s_1 & \Omega s_4 & s_3 \\ \Omega s_3 & s_4 & s_1 & \Omega s_2 \\ \Omega s_4 & \Omega s_3 & s_2 & s_1 \end{bmatrix} \tag{4}$$

where $s_1, \ldots, s_{n'} \in \mathcal{C}$ given in eq. (1) are the 2-dimensional vector representations of the 2-PPM information symbols. Evidently, $\Omega s \in \mathcal{C}$ given in eq. (1) whenever $s \in \mathcal{C}$ and the proposed code is shape-preserving with binary PPM.

*Proposition*: The proposed codes permit to achieve a full transmit diversity order with binary PPM. In particular, for $n = 2, 4, 8$ light sources (or transmit antennas), the codes are minimal delay, fully diverse and rate-1 codes.

*Proof*: The 2-PPM symbols verify the following relation:

$$\Omega s = -s + 1_2 \quad \forall\ s \in \mathcal{C} \tag{5}$$

The last equation shows that the pulse permutations introduced in eq. (3) and eq. (4) are directly related to the signal compliment when binary PPM constellations are used. This shows the similarity with [1] where the Alamouti code was adapted to FSO systems. On the other hand, expressing the PPM symbols in a 2-dimensional form as shown in eq. (1) and associating these symbols with the matrix representation given in eq. (4) allowed the generalization of the idea proposed in [1] to the more general family of orthogonal codes.

Following from eq. (5), the codewords in eq. (4) can be written as:

$$C(s_1, \ldots, s_{n'}) = C'(s_1, \ldots, s_{n'}) + C_0 \tag{6}$$

where, for a given value of $(n, n')$, the $2n \times n'$ matrix $C'$ can be obtained by replacing $\Omega$ by $-1$ in eq. (4). The matrix $C_0$ does not depend on the information symbols. For example, with $n = n' = 2$ and $n = n' = 4$, $C_0$ takes the following forms respectively:

$$C_0 = \begin{bmatrix} 0_2 & 0_2 \\ 1_2 & 0_2 \end{bmatrix} \ ; \ C_0 = \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 1_2 & 0_2 & 1_2 & 0_2 \\ 1_2 & 0_2 & 0_2 & 1_2 \\ 1_2 & 1_2 & 0_2 & 0_2 \end{bmatrix} \tag{7}$$

Based on the design criteria given in [10], the proposed code is fully diverse if the matrix $C(s_1, \ldots, s_{n'}) - C(s'_1, \ldots, s'_{n'})$ has a full rank for $(s_1, \ldots, s_{n'}) \neq (s'_1, \ldots, s'_{n'})$. Following from eq. (6) and from the linearity of the proposed code, it follows that full transmit diversity is achieved if the $2n \times n'$ matrix $C'(a_1, \ldots, a_{n'})$ has a full rank for $(a_1, \ldots, a_{n'}) \in \mathcal{A}^{n'} \setminus \{(0_2, \ldots, 0_2)\}$ where $\mathcal{A}$ denotes the set of all possible differences between two information vectors:

$$\mathcal{A} = \{s - s' \ ; \ s, s' \in \mathcal{C}\} = \{[0 \quad 0]^T \ , \ [1 \quad -1]^T \ , \ [-1 \quad 1]^T\} \tag{8}$$

Given that the rank of a matrix does not change when permuting its rows, then $C'(a_1, \ldots, a_{n'})$ has the same rank as the matrix:

$$C''(a_1, \ldots, a_{n'}) = \left[ (C'_1(a_1, \ldots, a_{n'}))^T \quad (C'_2(a_1, \ldots, a_{n'}))^T \right]^T$$

where $C'_i(a_1, \ldots, a_{n'})$ is the $n \times n'$ matrix composed from the rows of $C'(a_1, \ldots, a_{n'})$ having odd (resp. even) indices for $i = 1$ (resp. $i = 2$). From eq. (8), an element $a_i \in \mathcal{A}$ can be written as $a_i = a'_i [1 \quad -1]^T$ with $a'_i \in \{0, \pm 1\}$ for $i = 1, \ldots, n'$. Consequently:

$$C'_i(a_1, \ldots, a_{n'}) = (-1)^{i-1} C_{\text{orth}}(a'_1, \ldots, a'_{n'}) \tag{9}$$

where $C_{\text{orth}}(a'_1, \ldots, a'_{n'})$ corresponds to the $n \times n'$ codeword constructed from the real orthogonal design [5] and associated with the scalars $a'_1, \ldots, a'_{n'}$. Consequently, $\text{rank}(C_{\text{orth}}(a'_1, \ldots, a'_{n'})) = n$ unless when $a'_i = 0$ for $i = 1, \ldots, n'$. Therefore, $C''(a_1, \ldots, a_{n'})$ will have a full row rank unless when $a_i = 0_2$ for $i = 1, \ldots, n'$. As a conclusion, all the non-zero 2-dimensional vectors that result in a rank-deficient matrix $C(a_1, \ldots, a_{n'})$ do not belong to the set $\mathcal{A}$ given in eq. (8) and the proposed code permits to achieve a full transmit diversity order.

The proposed codes can be readily modified in order to be applied with the 1-dimensional OOK constellation given by $\mathcal{C} = \{0, 1\}$. In this case, from eq. (4), $s_1, \ldots, s_{n'} \in \mathcal{C}$ are scalars while $\Omega s_i$ must now be replaced by $-s_i + 1$ (the compliment of $s_i$) for $i = 1, \ldots, n'$.

Note that both the proposed scheme and the codes proposed in [8] are suitable for FSO and TH-UWB systems. In fact, both families of codes can be applied with 2, 4 and 8 transmitters without introducing any extension to the 2-PPM or OOK constellations. On the other hand, the proposed scheme presents the additional advantage of a symbol-by-symbol decodability

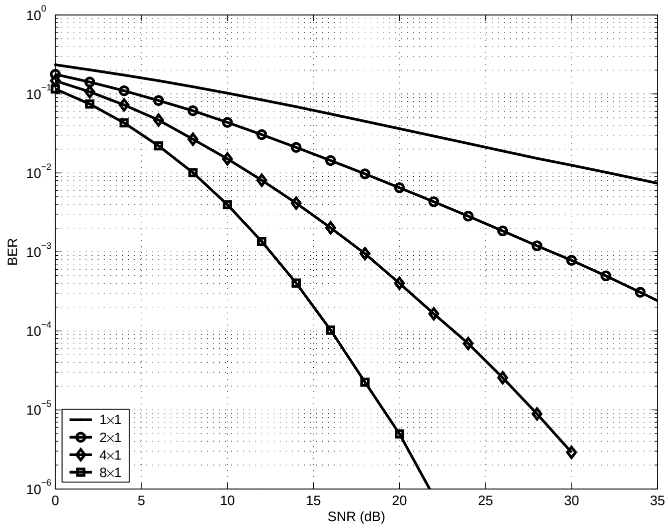


Fig. 1. Performance of the proposed orthogonal codes with binary PPM and MIMO FSO systems.

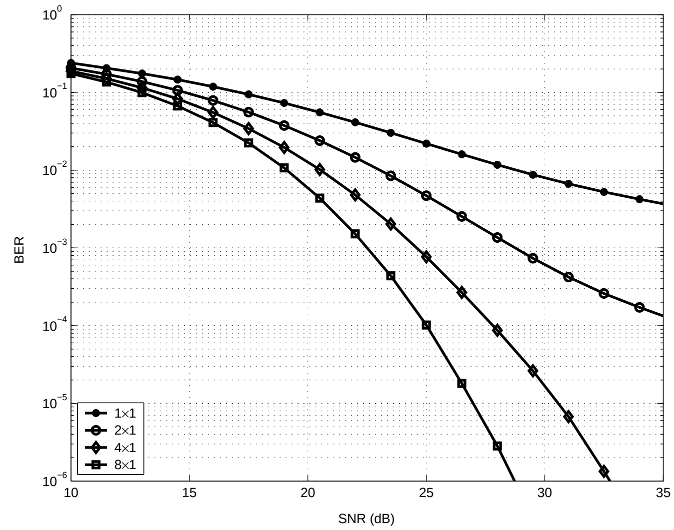


Fig. 2. Performance of the proposed orthogonal codes with binary PPM and TH-UWB over CM2 with a 1-finger Rake in the absence of IPI.

as will be shown in the next section. Note that the symbol-by-symbol decodability can be achieved even though the design is not orthogonal [11].

## IV. Maximum-Likelihood Decoding

Equation (2) can be written as:

$$\mathcal{X} = (I_{n'} \otimes H)\, \Phi(\Omega) S + \mathcal{N} \tag{10}$$

where $\mathcal{X}$ and $\mathcal{N}$ are $n'QM$-dimensional vectors given by: $\mathcal{X} = \text{vec}(X)$ and $\mathcal{N} = \text{vec}(N)$ respectively ($M=2$). $S$ is the $n'M$-dimensional vector obtained from the vertical concatenation of $s_1 \dots s_{n'}$. $\Phi(\Omega)$ is the $nn'M \times n'M$ matrix verifying: $\text{vec}(C) = \Phi(\Omega)S$. For example, with $n = n' = 2$:

$$\Phi(\Omega) = \left[ \begin{array}{cccc} I_M & \mathbf{0}_M & \mathbf{0}_M & I_M \\ \mathbf{0}_M & \Omega^T & I_M & \mathbf{0}_M \end{array} \right]^T \tag{11}$$

From eq. (10), the information vector $S$ can be determined based on the Maximum-Likelihood (ML) criterion: $\hat{S} = \arg\min_{S \in \mathcal{C}^{n'}} ||\mathcal{X} - (I_{n'} \otimes H)\, \Phi(\Omega) S||^2$. When $Q \geq P$, the decoding algorithm proposed in [8] can be applied in order to assure a ML detection of $S$. On the other hand, we will show in what follows that in the absence of IPI a simpler decoding technique based on linear processing can assure a ML detection.

Since $\text{vec}(C) = \Phi(\Omega)S = \Phi(\Omega)[s_1^T \cdots s_{n'}^T]^T$ and since the matrix $C'$ given in eq. (6) verifies $\text{vec}(C') = \Phi(-I_M)S$, then eq. (6) implies that:

$$\Phi(\Omega)S = \Phi(-I_M)S + I_0 = (\phi \otimes I_M)\, S + I_0 \tag{12}$$

where $I_0$ is the $nn'M$-dimensional vector given by $I_0 = \text{vec}(C_0)$ where $C_0$ is given in eq. (7). $\phi$ is the $nn' \times n'$ matrix that verifies the relation: $\text{vec}(C_{\text{orth}}(x_1, \dots, x_{n'})) = \phi[x_1 \cdots x_{n'}]^T$ where $x_1, \dots, x_{n'}$ are scalars. $\phi$ depends uniquely on the structure of the orthogonal codes [5] with $\phi^T \phi = n I_{n'}$.

In the absence of IPI, the $QM \times PM$ matrix $H$ in eq. (2) can be written as: $H = H' \otimes I_M$ where $H'$ is a $Q \times P$ matrix whose $(q,p)$-th element is equal to $h_{q,p}$ (the path gain between source $p$ and detector $q$). Consequently, combining eq. (10) and eq. (12) results in:

$$\mathcal{X} = [I_{n'} \otimes (H' \otimes I_M)]\, [(\phi \otimes I_M)\, S + I_0] + \mathcal{N} \tag{13}$$

Following from the properties of the Kronecker product, the last equation implies that:

$$\begin{aligned} \mathcal{Y} &\triangleq \mathcal{X} - [(I_{n'} \otimes H') \otimes I_M]\, I_0 && (14)\\ &= [(I_{n'} \otimes H')\, \phi \otimes I_M]\, S + \mathcal{N} && (15)\\ &\triangleq [\mathcal{H} \otimes I_M]\, S + \mathcal{N} && (16) \end{aligned}$$

From [5], the orthogonal code design corresponds to designing the matrix $\phi$ in such a way that the matrix $\mathcal{H} = (I_{n'} \otimes H')\, \phi$ verifies the relation: $\mathcal{H}^T \mathcal{H} = \left( \sum_{q=1}^{Q} \sum_{p=1}^{P} h_{q,p}^2 \right) I_{n'}$. Consequently, the constituent sub-vectors $s_1, \dots, s_{n'}$ of $S$ can be decoded independently according to:

$$p_i = \arg\max_{m=1,2} \left( \mathcal{Y}'_{2(i-1)+m} \right) \quad ; \quad s_i = e_{p_i} \quad ; \quad i = 1, \dots, n' \tag{17}$$

where $e_j$ is the $j$-th column of $I_M$ and $\mathcal{Y}'_j$ is the $j$-th component of the vector $\mathcal{Y}'$ given by:

$$\mathcal{Y}' = \left[ \mathcal{H}^T \otimes I_M \right] \mathcal{Y} \tag{18}$$

As a conclusion, eq. (14), eq. (17) and eq. (18) describe the detection procedures that must be performed in the absence of IPI. Note that in the presence of IPI, the relation $H = H' \otimes I_M$ does not hold and the above decoding procedures can not be applied. Moreover, following from the shape-preserving constraint and the constraint of symbol-by-symbol decodability, the rate of the proposed schemes can not exceed 1 symbol per channel use.

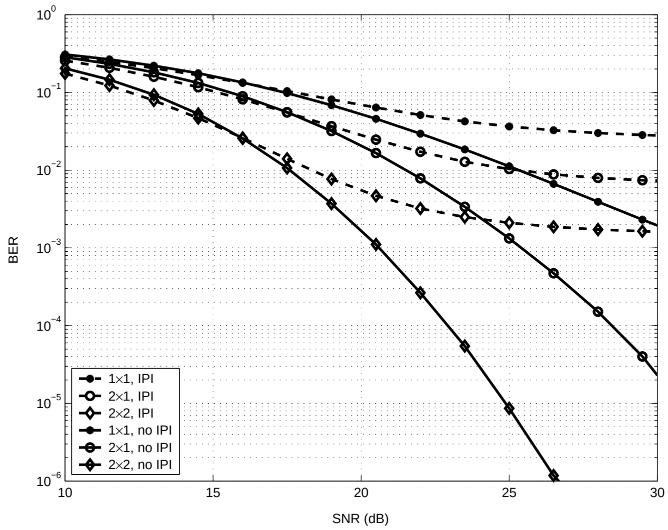


Fig. 3. The impact of IPI on ST-coded TH-UWB systems over CM2 with a 5-finger Rake and binary PPM. In the absence of IPI, the linear decoding strategy proposed in section IV is applied while the non-linear algorithm proposed in [8] is applied in the presence of IPI.

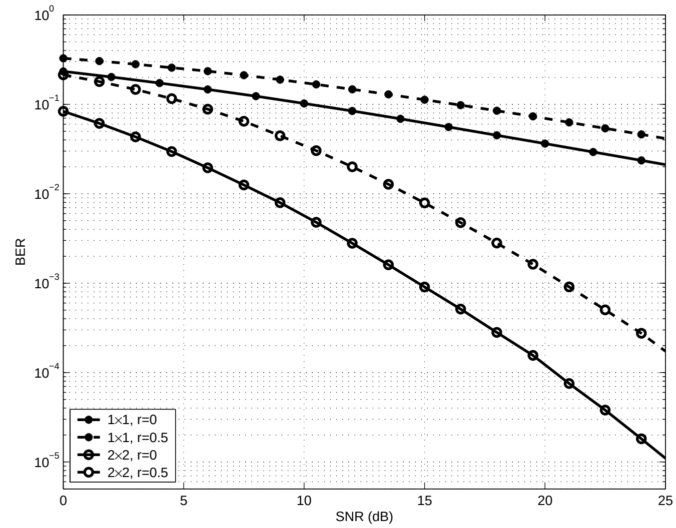


Fig. 4. The impact of IPI on ST-coded MIMO-FSO systems. Nonorthogonal PPM constellations are used and the scalar $r$ stands for the autocorrelation, of the light waveform, evaluated at the temporal separation between two pulse positions. $r = 0$ stands for the absence of IPI.

## V. Simulations and results

Fig. 1 shows the performance of SISO FSO systems and $n \times 1$ FSO systems using the minimal-delay $n \times n$ codes for $n = 2, 4, 8$. Flat fading channels are considered (no IPI) and, as in [2], the channel irradiances are drawn from an exponential distribution whose mean is equal to 1. Simulations of TH-UWB systems over the IEEE 802.15.3a channel model recommendation CM2 [9] are shown in Fig. 2 in the absence of IPI. High performance gains are evident in both cases.

The impact of IPI on TH-UWB is shown in Fig. 3. In this case, the modulation delay is chosen to be equal to 0.5 ns which is much smaller than the channel delay spread of the CM2 channels. In this case, the decoder proposed in [8] is applied. Results show the utility of the proposed schemes in reducing the error floors induced by IPI. Similar results are obtained in Fig. 4 with FSO systems. In this case, the MIMO channel is supposed to be flat while the separation between the PPM positions is taken to be smaller than the width of the light pulses.

## VI. Conclusion

By replacing the phase rotations with pulse permutations, the orthogonal ST codes were extended to binary PPM. MIMO FSO and MIMO TH-UWB systems can now take advantage from the unique properties of the orthogonal codes without introducing any additional constellation extension. In other words, full transmit diversity can be achieved while conveying the information only through the time delays of the modulated light pulses transmitted from the different light sources in MIMO FSO systems. In the same way, the extension of the existing single-antenna TH-UWB systems to the MIMO scenarios will not necessitate additional constraints on the RF circuitry to control the phase or the amplitude of the very low duty cycle UWB pulses. A similar argument holds for MIMO FSO and MIMO TH-UWB systems with OOK.

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