A Space-Time Coded MIMO TH-UWB Transceiver with Binary Pulse Position Modulation

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***Abstract*— In this paper, we propose a new Multiple-Input-Multiple-Output (MIMO) transceiver for Time-Hopping Ultra-Wideband (TH-UWB) communications. We consider the problem of Space-Time (ST) coding with binary Pulse Position Modulation (PPM) and we propose the first known family of rate-1 ST codes that can be associated with binary PPM without introducing any additional constellation extension. We prove that the proposed encoding scheme can achieve a full transmit diversity order with 2^k transmit antennas. At the receiver side, we propose a Maximum-Likelihood (ML) decoder that is adapted to the structure of the considered multi-dimensional constellation that does not have the structure of a lattice.**

***Index Terms*— UWB, Space-Time, MIMO, PPM, ML decoding.**

I. Introduction

THE literature of ST coding is huge [1]–[4]. However, all the known ST coding schemes introduce a constellation extension when associated with PPM. Consider for example the orthogonal codes [2]. The entries of the codewords are equal to $\pm s_i$ or $\pm s_i^*$ where $s_1, \dots, s_n$ stand for the information symbols and n is the number of transmit antennas. While these codes are shape preserving with QAM, they introduce a constellation extension when associated with PPM since $-s$ and s can not be both PPM symbols simultaneously. In the same way, the codes proposed in [5], [6] for TH-UWB communications necessitate rotating the phase or amplifying the amplitude of the transmitted symbols and, consequently, they are not adapted to PPM.

Instead of adopting the classical approach of constructing ST block codes over the hypercubes carved from lattices, we take advantage of the structure of the binary PPM constellation in order to construct a 2-PPM-specific code. This code keeps the natural advantages of carrier-less impulse radio TH-UWB where the information is conveyed only by the time delays of the very short transmitted pulses.

At the receiver side, we consider the problem of ML decoding with the multi-dimensional 2-PPM constellation. This constellation is sparse and the different components of the transmitted information vectors are not independent. Consequently, the lattice decoders proposed in the literature [7], [8] can not be associated with this constellation. In this work, we propose a convenient ML decoding scheme that avoids the convergence towards a non-valid point (that corresponds to a lattice point that does not belong to the multi-dimensional 2-PPM constellation).

Notations: 0_n and 1_n correspond to the n-dimensional vectors whose components are equal to 0 and 1 respectively. $\text{vec}(X)$ stacks the columns of the matrix X vertically. $\otimes$ stands for the Kronecker product.

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II. System Model

A binary PPM constellation is a 2-dimensional constellation where the information symbols are represented by 2-dimensional vectors belonging to the set:

$$\mathcal{C} = \{[1 \quad 0]^T, [0 \quad 1]^T\} \tag{1}$$

where the non-zero component of each vector indicates the presence of a UWB pulse.

Consider a MIMO TH-UWB system equipped with P transmit antennas and Q receive antennas. Each receive antenna is equipped with a L-finger Rake. At each finger of the Rake, a bank of $M = 2$ correlators is used in order to separate the different components of the 2-dimensional transmitted signals. For a single-user scenario in the absence of inter-symbol-interference, the linear dependence between the baseband inputs and outputs of the channel can be expressed as:

$$Y = HC + N \tag{2}$$

where C is the $PM \times T$ codeword whose $((p-1)M + m, t)$-th entry corresponds to the amplitude of the pulse (if any) transmitted at the m-th position of the p-th antenna during the t-th symbol duration for $p = 1, \dots, P$, $m = 1, \dots, M = 2$ and $t = 1, \dots, T$. The matrices Y and N are $QLM \times T$ matrices corresponding to the decision variables and the noise terms respectively.

H is the $QLM \times PM$ channel matrix given by $H = [H_1^T \cdots H_Q^T]^T$ where $H_q = [H_{q,1}^T \cdots H_{q,L}^T]^T$ for $q = 1, \dots, Q$. The matrix $H_{q,l}$ is given by $H_{q,l} = [H_{q,l,1} \cdots H_{q,l,P}]$ for $l = 1, \dots, L$. $H_{q,l,p}$ is a $M \times M$ matrix for $p = 1, \dots, P$. The (m, m')-th element of $H_{q,l,p}$ corresponds to the impact of the signal transmitted during the m'-th position of the p-th antenna on the m-th correlator (corresponding to the m-th position) placed after the l-th Rake finger of the q-th receive antenna. This element is given by:

$$H_{q,l,p}(m, m') = h_{q,p}((m - m')\delta + \Delta_l) \tag{3}$$

where δ stands for the modulation delay and Δ_l for the l-th finger delay. In what follows, we consider a Rake that combines the first arriving multi-path components and we fix $\Delta_l = (l–1)MT_w$ where T_w stands for the duration of the UWB pulse (with $\delta \geq T_w$). Designate by $g_{q,p}(t)$ the convolution of the pulse waveform $w(t)$ with the impulse response of the frequency selective channel between antennas p and q. In this case, $h_{q,p}(\tau) = \int_0^{T_f} g_{q,p}(t)w(t-\tau)\mathrm{d}t$ where T_f stands for the average separation between two consecutive pulses.

III. Code Construction

For binary PPM with $n = P$ transmit antennas, the minimal-delay codewords are represented by the matrices of dimensions $2n \times n$ having the following structure:

$$C(s_1, \dots, s_n) = \begin{bmatrix} s_1 & s_2 & \cdots & s_n \\ \Omega s_n & s_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & s_2 \\ \Omega s_2 & \cdots & \Omega s_n & s_1 \end{bmatrix} \quad (4)$$

where $s_1, \dots, s_n \in \mathcal{C}$ given in eq. (1) and Ω is the 2×2 cyclic permutation matrix given by:

$$\Omega = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (5)$$

It can be directly seen that the proposed code does not introduce any constellation extension with binary PPM since $\Omega s \in \mathcal{C}$ given in eq. (1) whenever $s \in \mathcal{C}$.

Proposition: For binary PPM, the rate-1 code proposed in eq. (4) permits to achieve a full transmit diversity order with $n = 2^k$ transmit antennas for all integer values of k.

Proof: Denote by $\mathcal{A}$ the set of all possible differences between two information vectors:

$$\mathcal{A} = \{s - s' \ ; \ s, s' \in \mathcal{C}\} = \{[0 \ \ 0]^T, [1 \ \ -1]^T, [-1 \ \ 1]^T\} \quad (6)$$

The transmit diversity is achieved because of this particular structure of $\mathcal{A}$. In the Appendix, we prove that all the non-zero 2-dimensional vectors that result in a rank-deficient matrix C do not belong to the set $\mathcal{A}$ given in eq. (6) implying that the code is fully diverse.

IV. ML decoding algorithm

Equation (2) can be written as:

$$y = \mathrm{vec}(Y) = (I_n \otimes H)\Phi[s_1^T \ \cdots \ s_n^T]^T + \mathrm{vec}(N) \triangleq \mathcal{H}s + n \quad (7)$$

where Φ is the $n^2M \times nM$ matrix verifying $\mathrm{vec}(C(s_1, \dots, s_n)) = \Phi s$ (note that $P = n$ and $M = 2$).

For 2-PPM, the information vector s is composed of P sub-vectors that are equal to the columns of the 2×2 identity matrix and consequently the coordinates of s are not independent. Moreover, the generated $2P$-dimensional lattice has a cardinality of 2^{2P} since the amplitude in each modulation position can take 2 values. However, among these points only 2^P points are valid. Therefore, applying the sphere decoding algorithms [7], [8] will result in non-valid points since the $2P$ components of s must not be spanned independently. In what follows, we propose an adapted ML decoding algorithm for this sparse multi-dimensional constellation.

In what follows s will be seen as composed of P layers having 2 dimensions each. The algorithm that we propose is inspired from the Schnorr-Euchner enumeration strategy proposed in [8] for QAM. Two major modifications are applied on this strategy to render it suitable for 2-PPM. First, a multi-dimensional zero-forcing decision feedback equalization (ZF-DFE) must be applied between the different layers (and not the different components). Second, a multi-dimensional span is applied to determine the order with which the two candidate sub-vectors of each layer must be tested.

Equation (7) can be written as $z = Q^T y = Rs + Q^T n$. Q is a unitary matrix and R an upper triangular matrix (with positive diagonal elements) obtained by applying a QR decomposition on the matrix $\mathcal{H}$. Let $V = R^{-1}$ and designate by $X^{(p,p')}$ the $M \times M$ matrix composed of the elements $X_{i,j}$ of the $PM \times PM$ matrix X for $i = (p-1)M + 1, \dots, pM$ and $j = (p'-1)M + 1, \dots, p'M$ for $p, p' = 1, \dots, P$. The m-th column of X will be denoted by $X_{:,m}$.

Denote by $e^{(k)}$ the 2-dimensional vector obtained by applying ZF on the layers $1, \dots, k-1, k+1, \dots, P$ and by taking hard decisions on the layers $k+1, \dots, P$. The influence of the k-th layer on the squared Euclidean distance between the received decision vector (considered in the transmitted signal subspace) and the candidate output of the algorithm when a pulse is transmitted at the m-th position is given by:

$$d_k^2 = \left\| R^{(k,k)} \left(e^{(k)} - I_{:,m} \right) \right\|^2 = \left\| E^{(k)} - R^{(k,k)}_{:,m} \right\|^2 \quad (8)$$

where $E^{(k)} \triangleq R^{(k,k)} e^{(k)}$ and $I_{:,m}$ corresponds to the m-th column of the 2×2 identity matrix I. Therefore, at the k-th layer, the algorithm must first test the symbol corresponding to the modulation position given by:

$$\mathrm{argmin}_{m=1,2} \left\{ -2(R^{(k,k)}_{:,m})^T E^{(k)} + (R^{(k,k)}_{:,m})^T R^{(k,k)}_{:,m} \right\} \quad (9)$$

The decoding algorithm is given after fixing the following notations. ρ is a 2-dimensional vectors. e and E are $2P \times P$ matrices. $X^{(p)}$ corresponds to the elements $2(p-1)+1, \dots, 2p$ of the $2P$-dimensional vector X for $p = 1, \dots, P$.

Decoding Algorithm (Input: z, R, P, C. Output: $\hat{s}$)

Step 1: Set $k = P + 1$, $dist(k) = 0$, $V = R^{-1}$, $\rho = 0_2$, $f = 0_P$, $bestdist = C$ (sphere squared radius)

Step 2: $newdist = dist(k) + \rho^T \rho$,
if $(newdist < bestdist) \& (k \neq 1)$ go to Step 3
else go to Step 4 endif.

Step 3: if k=P+1, $e_{:,k-1} = Vz$ else
for $i = 1, \dots, k-1$,
$e^{(i)}_{:,k-1} = e^{(i)}_{:,k} - V^{(i,k)}\rho$ endfor endif.
$k = k-1$, $dist(k) = newdist$, $E^{(k)}_{:,k} = R^{(k,k)} e^{(k)}_{:,k}$,

$$pos(k) = \mathrm{argmin}_{m=1,2} \left\{ -2(R^{(k,k)}_{:,m})^T E^{(k)} + (R^{(k,k)}_{:,m})^T R^{(k,k)}_{:,m} \right\}$$

$x_{:,k} = I_{:,pos(k)}$, $\rho = E^{(k)}_{:,k} - R^{(k,k)}_{:,pos(k)}$ go to Step 2.

Step 4: if $newdist < bestdist$, for $i = 1, \dots, P$, $\hat{s}^{(i)} = x_{:,i}$ endfor, $bestdist = newdist$
elseif $k = P$ terminate else $k = k + 1$ endif.
if $f(k) = 1$ go to Step 4 endif,
$pos(k) = pos(k) \mod 2 + 1$, $x_{:,k} = I_{:,pos(k)}$,
$f(k) = 1$, $\rho = E^{(k)}_{:,k} - R^{(k,k)}_{:,pos(k)}$ go to Step 2.

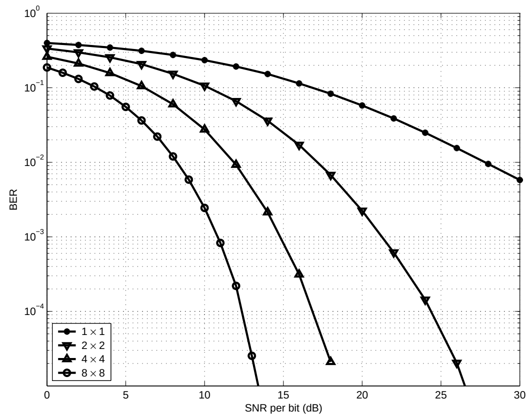

Fig. 1. Performance of the proposed code with n transmit antennas,n receive antennas and a 1-finger Rake for $n = 2, 4, 8$.

V. Simulations and Results

Simulations show the performance over the IEEE 802.15.3a channel model recommendation CM2 [9]. The second derivative of the Gaussian pulse with a duration of 0.5 ns is used. The symbol duration is chosen to be larger than the channel delay spread in order to eliminate the inter-symbol-interference.

Fig. 1 compares the performance of single-antenna systems and of the ST coded MIMO UWB systems with $n = 2^k$ transmit antennas for $k = 1, 2, 3$. The receiver is composed of n antennas each equipped with a 1-finger Rake. Results show the enhanced diversity order and the high performance levels achieved by the proposed coding scheme.

VI. Conclusion

We discussed issues related to the encoding and decoding of impulsive signals composed of time-shifted pulses. Unlike all the existing MIMO UWB systems, the proposed solution is appealing since it does not necessitate additional constraints on the RF circuitry to control the phase or the amplitude of the very low duty cycle sub-nanosecond pulses. At the receiver side, the multi-dimensional decoder assures a fast and efficient separation between the transmitted data streams.

Appendix

For any element $s \in \mathcal{C}$, we have $\Omega s = -s + 1_2$. Consequently, eq. (4) can be written as:

$$C(s_1, \dots, s_n) \triangleq C'(s_1, \dots, s_n) + C_0 \tag{10}$$

$$= \begin{bmatrix} s_1 & s_2 & \cdots & s_n \\ -s_n & s_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & s_2 \\ -s_2 & \cdots & -s_n & s_1 \end{bmatrix} + \begin{bmatrix} 0_2 & 0_2 & \cdots & 0_2 \\ 1_2 & 0_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0_2 \\ 1_2 & \cdots & 1_2 & 0_2 \end{bmatrix} \tag{11}$$

The proposed code is fully diverse if the matrix $C(s_1, \dots, s_n) - C(s'_1, \dots, s'_n)$ has a full rank for $(s_1, \dots, s_n) \neq (s'_1, \dots, s'_n)$. Following from eq. (10) and from the linearity of the proposed code, it follows that full transmit diversity is achieved if the $2n \times n$ matrix $C'(a_1, \dots, a_n)$ has a full rank for $(a_1, \dots, a_n) \in \mathcal{A}^n \backslash \{(0_2, \dots, 0_2)\}$ where $\mathcal{A}$ is given in eq. (6).

The matrix $C'(a_1, \dots, a_n)$ can be written as:

$$C'(a_1, \dots, a_n) = \left[\Gamma C''(a'_1, \dots, a'_n)\Gamma^{-1}\right] \otimes [1 \quad -1]^T \tag{12}$$

where, from eq. (6), an element $a_i \in \mathcal{A}$ can be written as $a_i = a'_i[1 \quad -1]^T$ with $a'_i \in \{0, \pm 1\}$ for $i = 1, \dots, n$. $\Gamma = \text{diag}([(-1)^{\frac{n-k}{n}}]_{k=1}^n)$ and $C''(a'_1, \dots, a'_n)$ is given by:

$$C''(a'_1, \dots, a'_n) = \begin{bmatrix} a'_1 & \omega_{2n} a'_2 & \cdots & \omega_{2n}^{n-1} a'_n \\ \omega_{2n}^{n-1} a'_n & a'_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \omega_{2n} a'_2 \\ \omega_{2n} a'_2 & \cdots & \omega_{2n}^{n-1} a'_n & a'_1 \end{bmatrix} \tag{13}$$

where $\omega_k = \exp\left(\frac{2\pi i}{k}\right)$ is the k-th primitive root of unity.

Equation (13) corresponds to a special case of the ST codes proposed in [10]. To prove that the code is fully diverse, we must prove that $\{1, \omega_{2n}, \dots, \omega_{2n}^{n-1}\}$ are algebraically independent over $\mathbb{Q}$. In this case, $C''(a'_1, \dots, a'_n)$ has full rank unless when $a'_1 = \cdots = a'_n = 0$. Consequently, $C(a_1, \dots, a_n)$ will have full rank unless when $a_1 = \cdots = a_n = 0_2$ and the proposed code will achieve full transmit diversity with 2-PPM.

$\mathbb{Q}(\omega_{2n})$ has degree $\varphi(2n)$ over $\mathbb{Q}$ where $\varphi(.)$ stands for Euler's totient function. $\{1, \dots, \omega_{2n}^{n-1}\}$ are algebraically independent if $\varphi(2n) \geq n$. On the other hand, $2n$ is not relatively prime with at least $n-1$ elements of the set $\{1, \dots, 2n-1\}$. These numbers are given by $S = \{2k\}_{k=0}^{n-1}$ and consequently $\varphi(2n) \leq n$. This implies that algebraic independence is possible only when $\varphi(2n) = n$. The only values of n that verify this relation are the powers of 2. In fact, if in the prime decomposition of n appears a prime number that is not equal to 2, then this number (in addition to the elements of S) are not relatively prime with $2n$ and $\varphi(2n) < n$. For $n = 2^k$, all the elements that are relatively prime with $2n$ are included in S and, consequently, $\varphi(2n) = n$ is verified only with $n = 2^k$.

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